Lecture 4

Part 1: Finish Geometrical Optics
Part 2: Physical Optics

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Aberrations

- In optical systems
- Description in terms of Zernike polynomials
- Aberrations due to atmospheric turbulence

- Based on slides by Brian Bauman, LLNL and UCSC, and Gary Chanan, UCI

Optical aberrations: first order and third order Taylor expansions

- ***sin θ*** **terms in Snell's law can be expanded in power series**

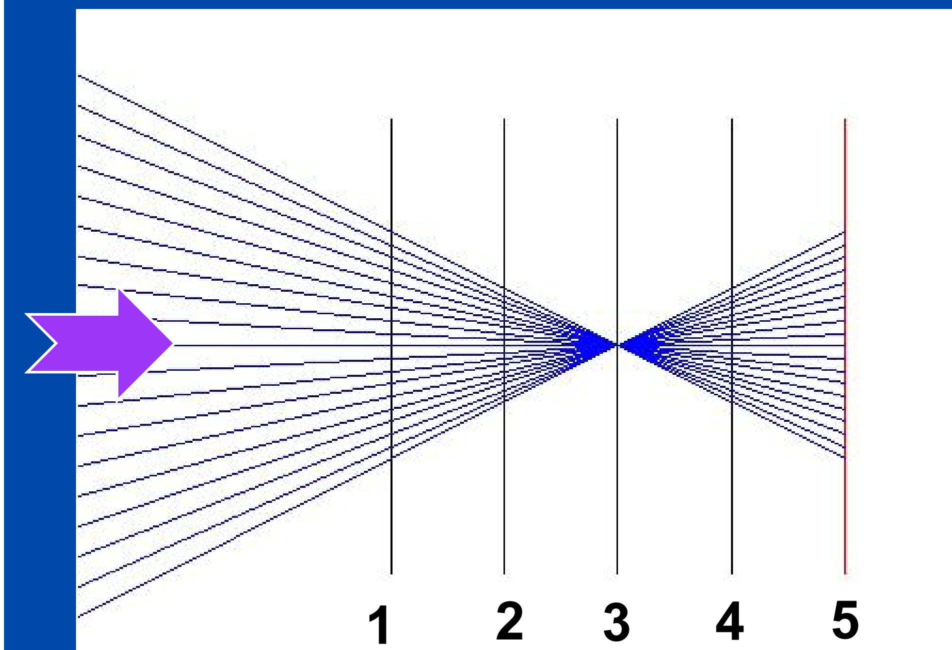
$$n \sin\theta = n' \sin\theta'$$

$$n\,(\theta - \theta^3/3! + \theta^5/5! + \ldots) = n'\,(\theta' - \theta'^3/3! + \theta'^5/5! + \ldots)$$

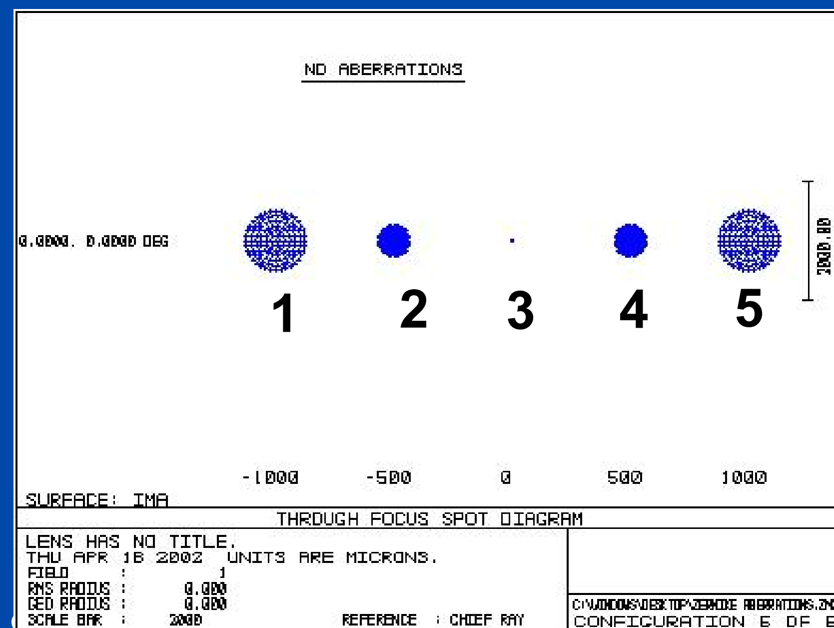
- **Paraxial ray approximation: keep only *θ* terms (first order optics; rays propagate nearly along optical axis)**
  - Piston, tilt, defocus
- **Third order aberrations: result from adding θ^3 terms**
  - Spherical aberration, coma, astigmatism, ...

Different ways to illustrate optical aberrations

Side view of a fan of rays
(No aberrations)

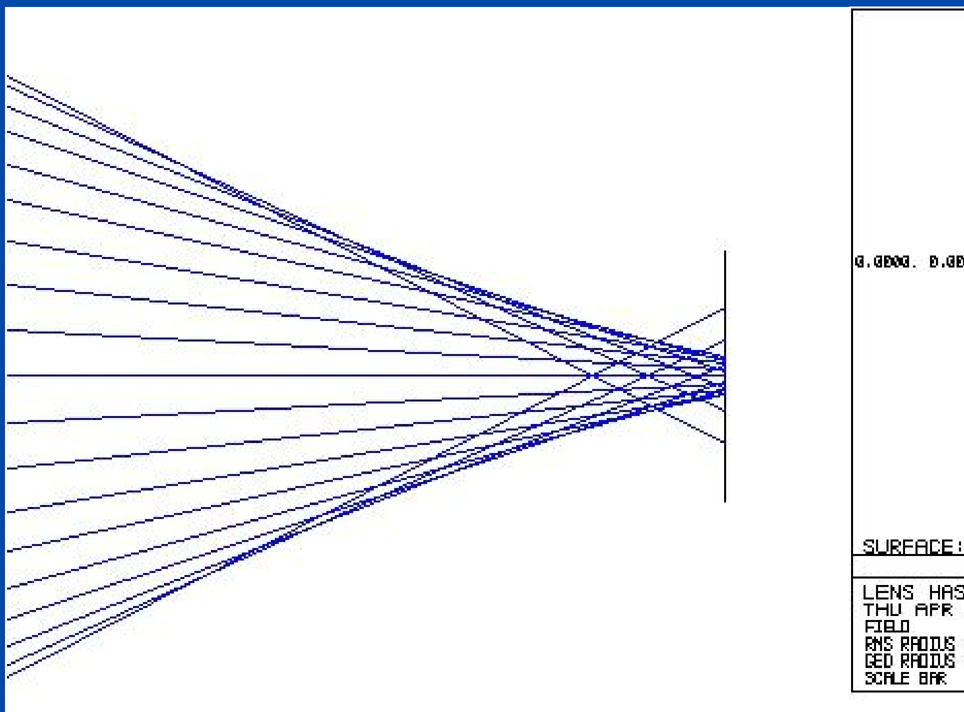


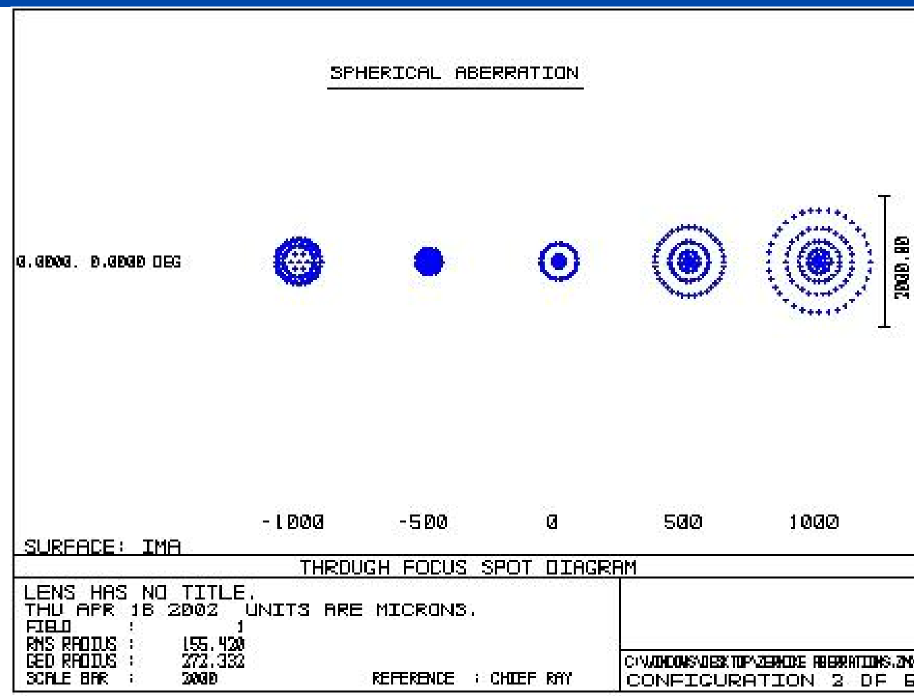
“Spot diagram”: Image at different focus positions



strike hypothetical detector

Spherical aberration

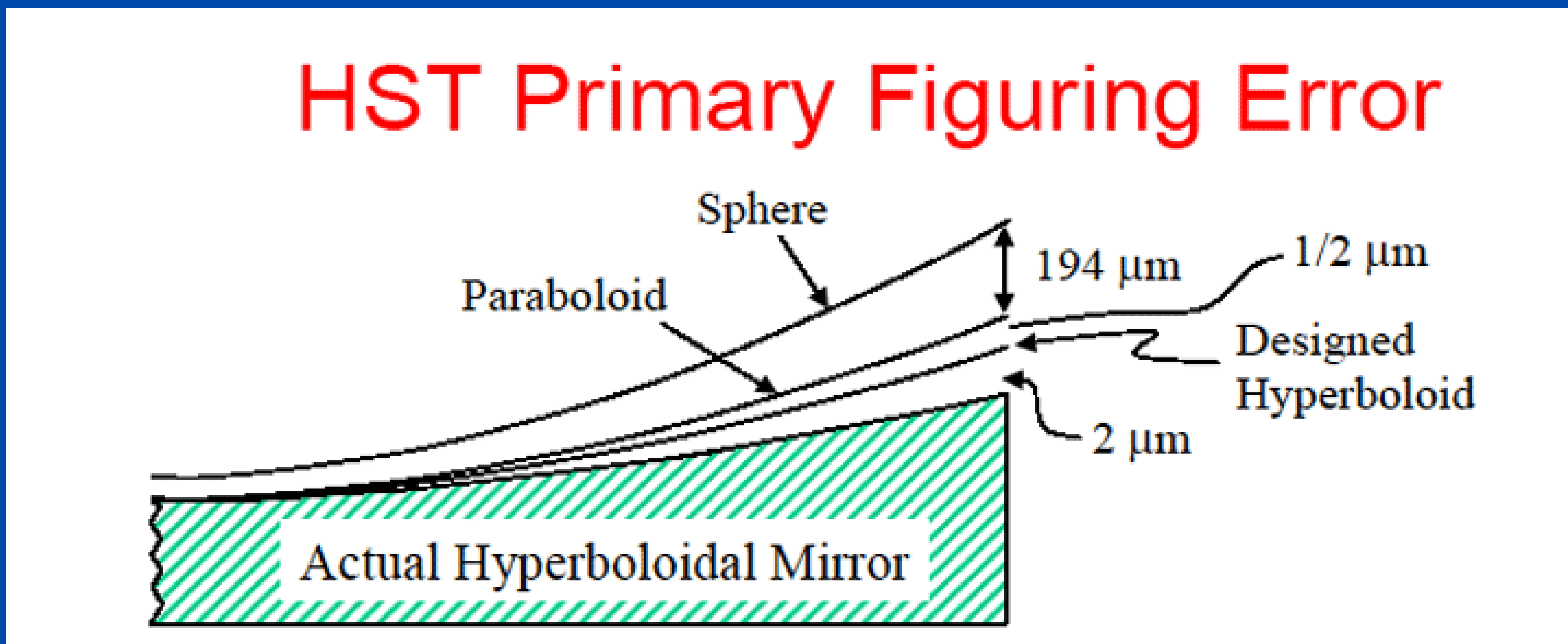




Rays from a spherically aberrated wavefront focus at different planes

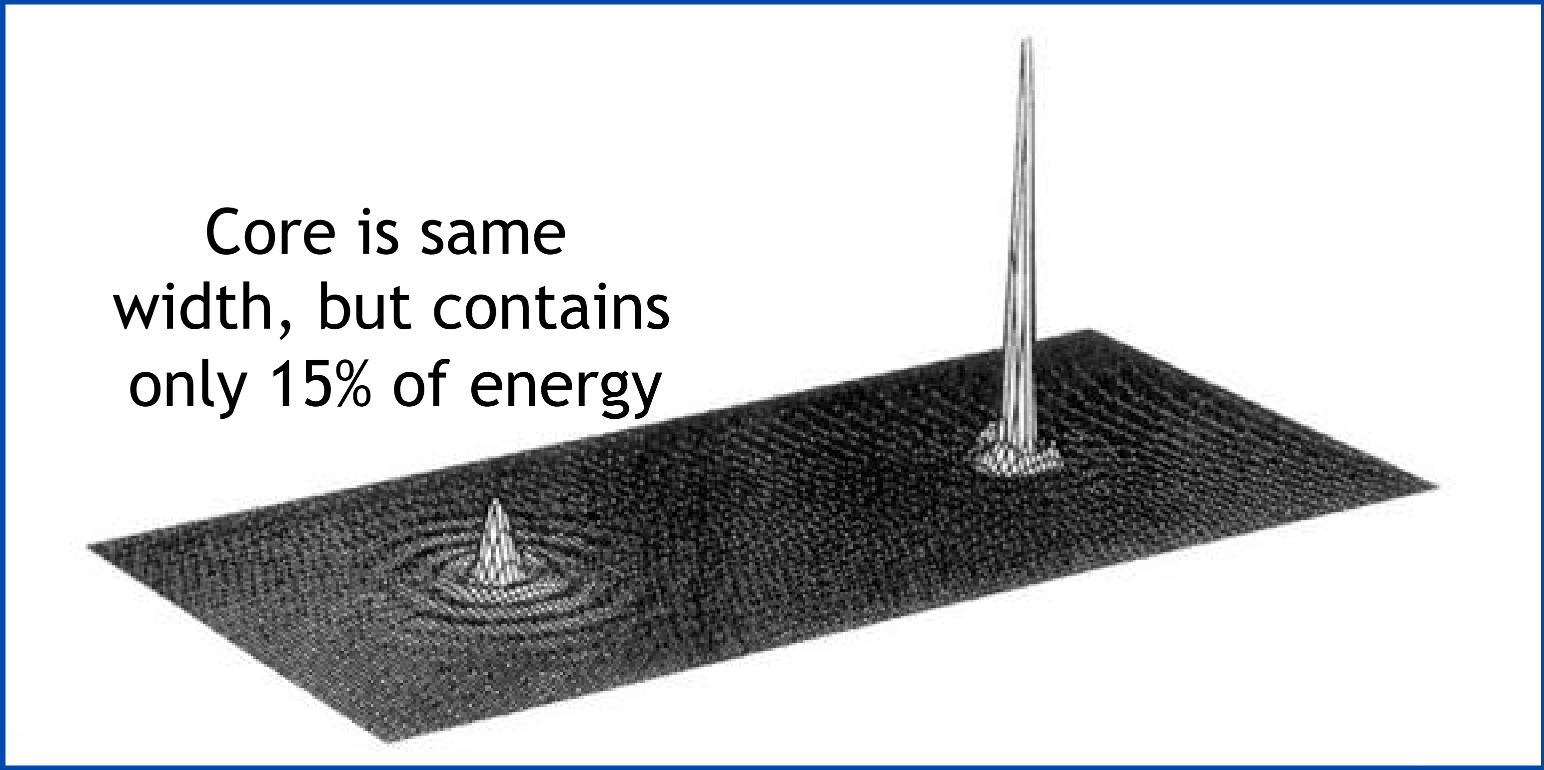
Through-focus spot diagram for spherical aberration

Hubble Space Telescope suffered from Spherical Aberration



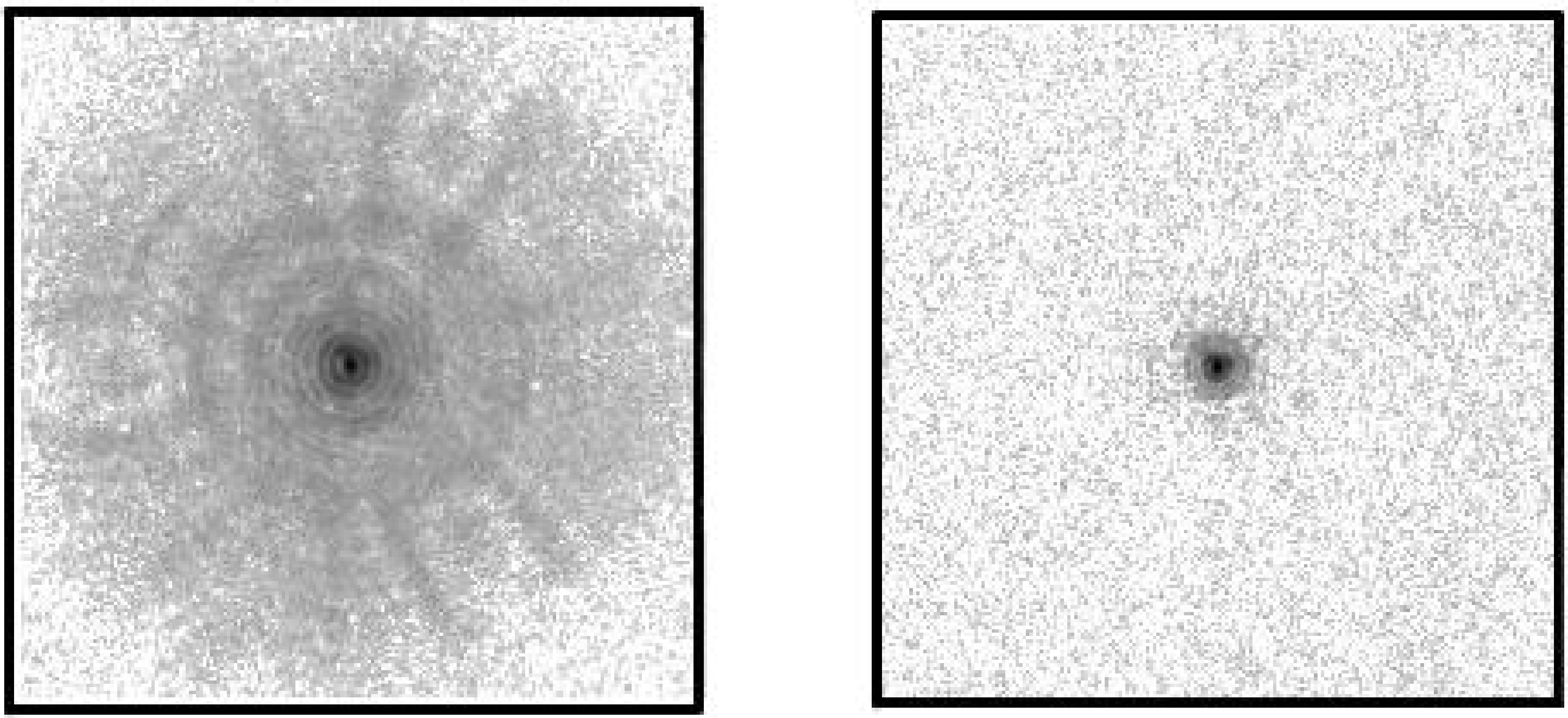
- **In a Cassegrain telescope, the hyperboloid of the primary mirror must match the specs of the secondary mirror. For HST they didn't match.**

HST Point Spread Function (image of a point source)



Before COSTAR fix **After COSTAR fix**

Point spread functions before and after spherical aberration was corrected

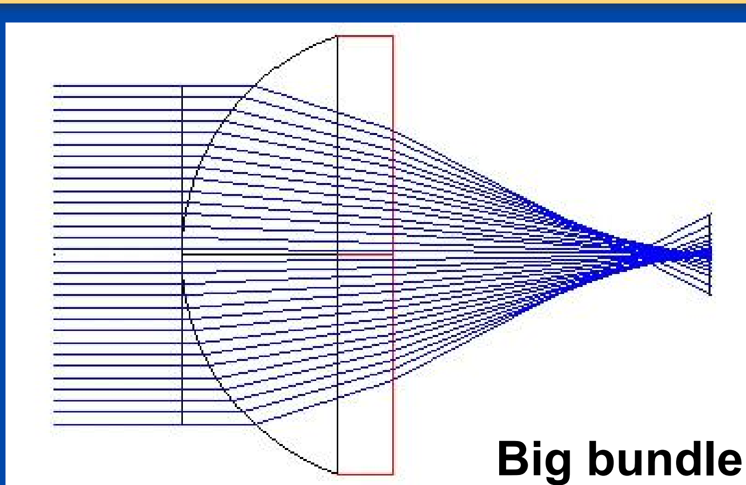


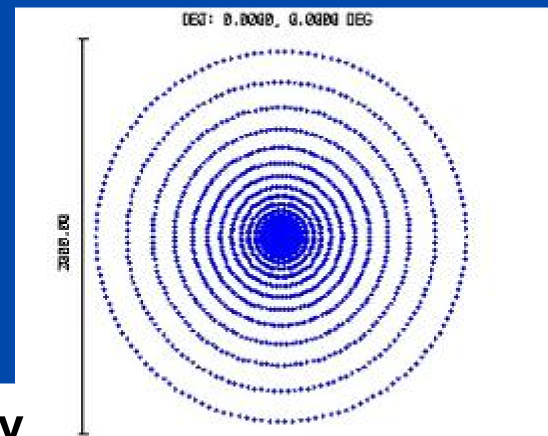
Central peak of uncorrected image (left) contains only 15% of central peak energy in corrected image (right)

Spherical aberration as "the mother of all other aberrations"

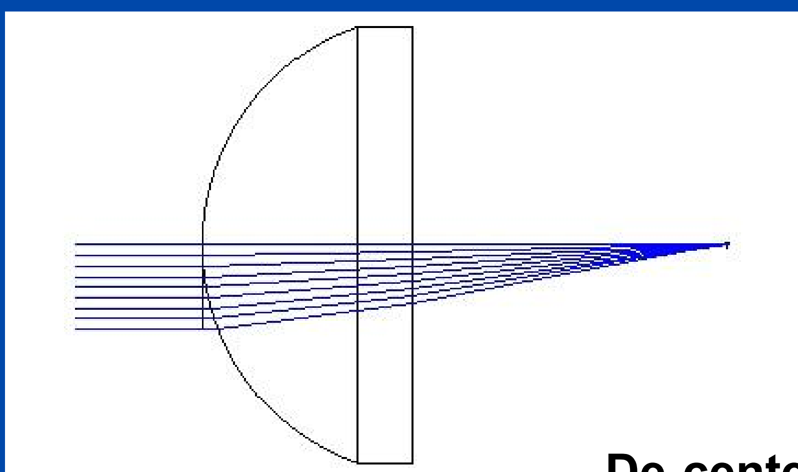
- Coma and astigmatism can be thought of as the aberrations from a de-centered bundle of spherically aberrated rays
- Ray bundle on axis shows spherical aberration only
- Ray bundle slightly de-centered shows coma
- Ray bundle more de-centered shows astigmatism
- All generated from subsets of a larger centered bundle of spherically aberrated rays
  - (diagrams follow)

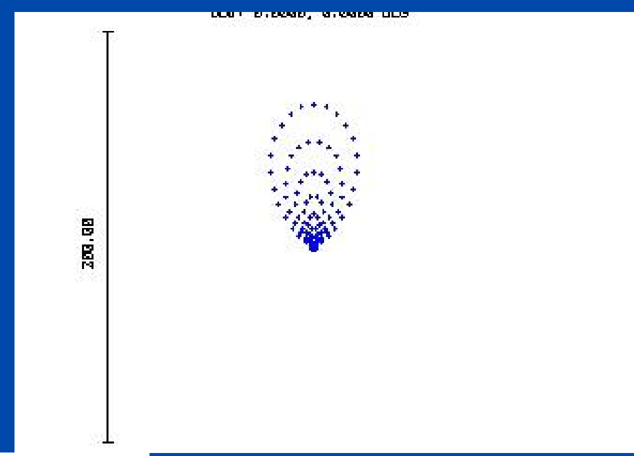
Spherical aberration as the mother of coma





Big bundle of spherically aberrated rays

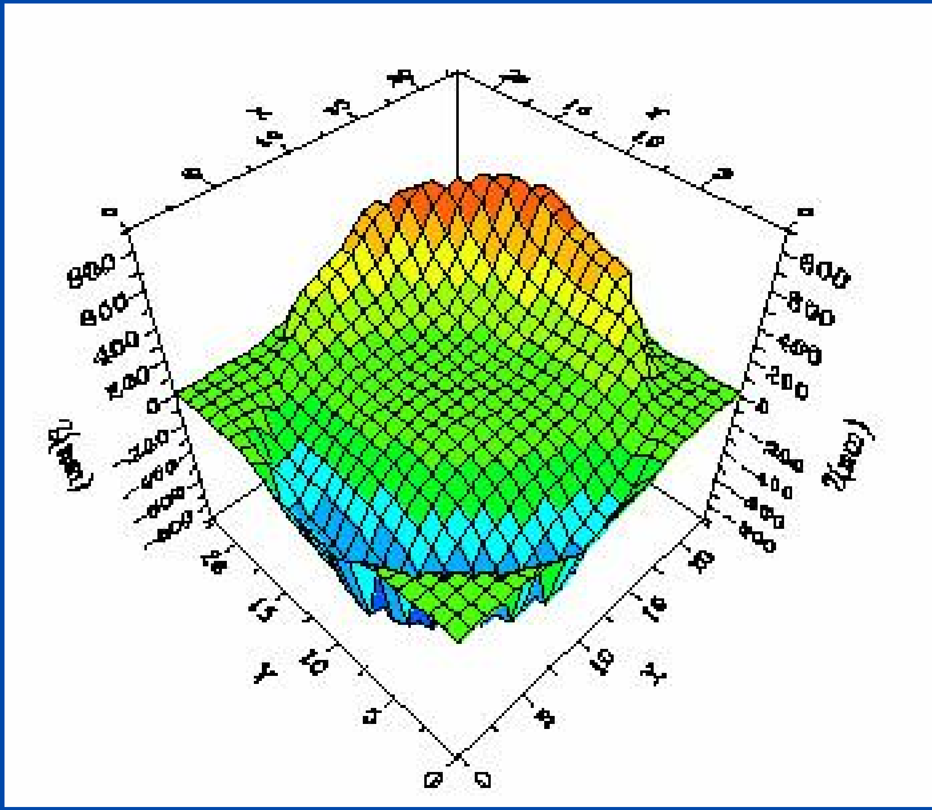




De-centered subset of rays produces coma

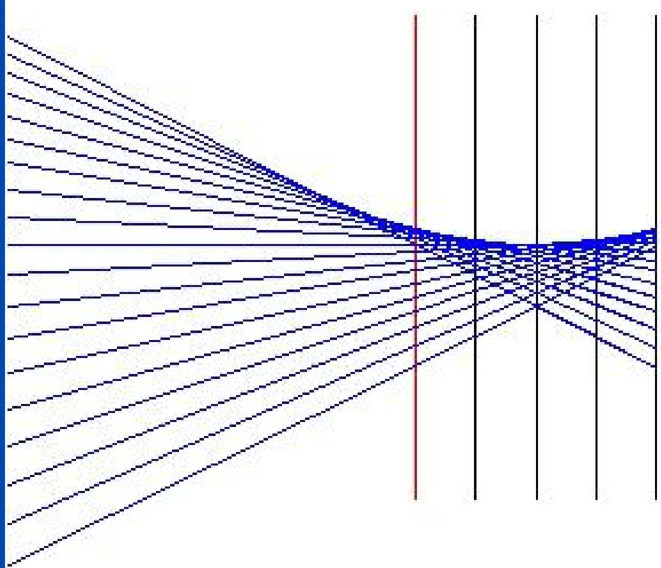
Coma

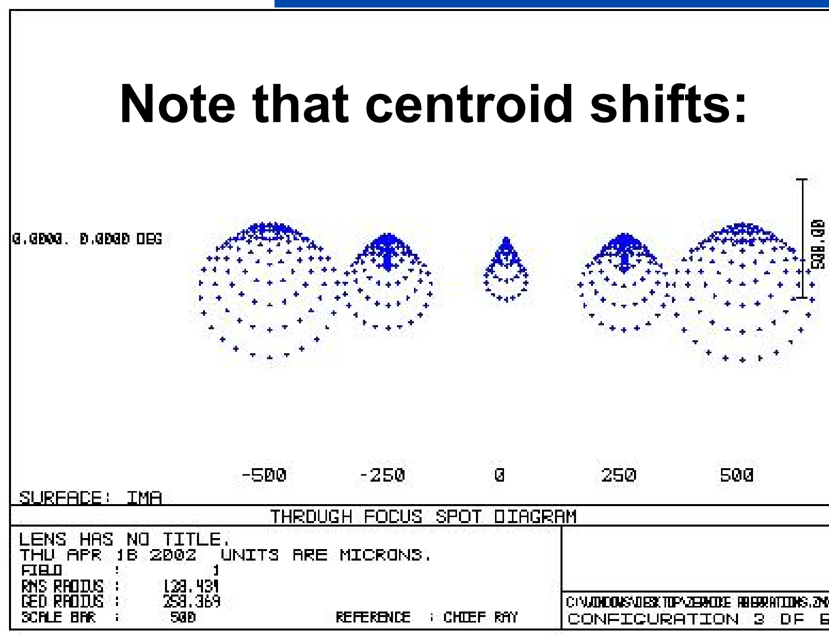
- "Comet"-shaped spot
- Chief ray is at apex of coma pattern
- Centroid is shifted from chief ray!
- Centroid shifts with change in focus!



Wavefront

Coma

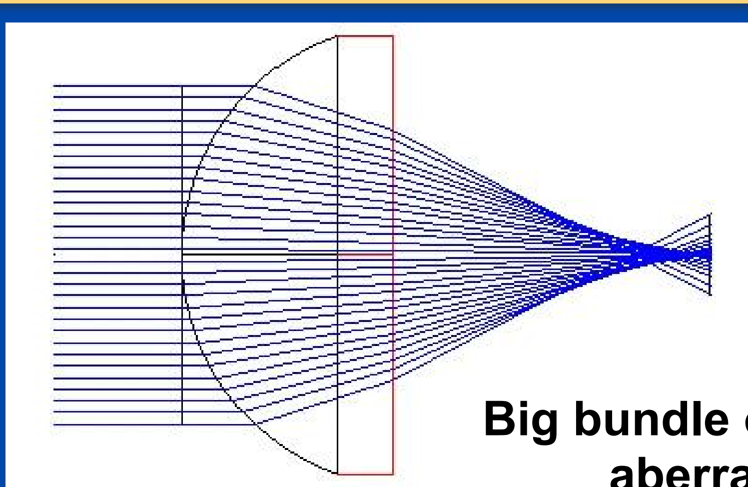


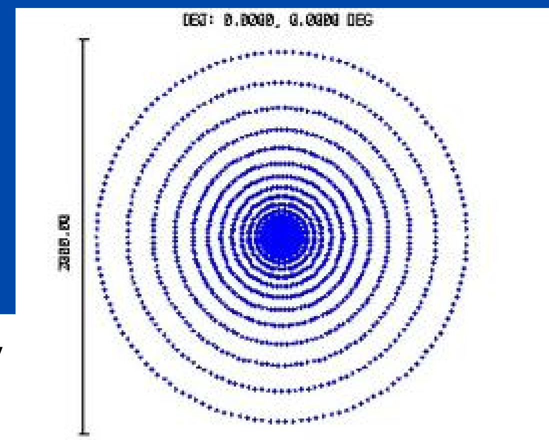


Rays from a comatic wavefront

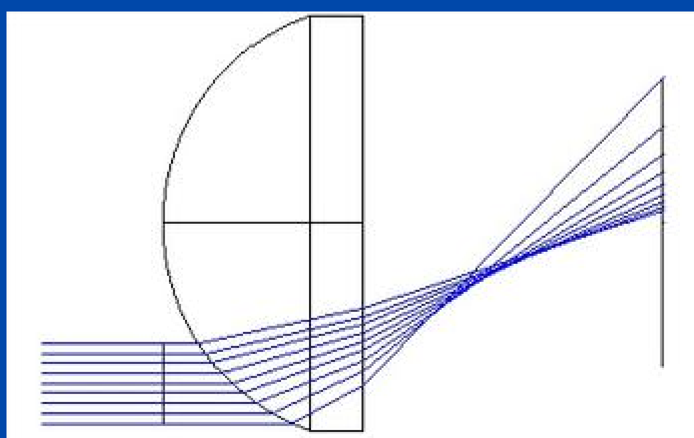
Through-focus spot diagram for coma

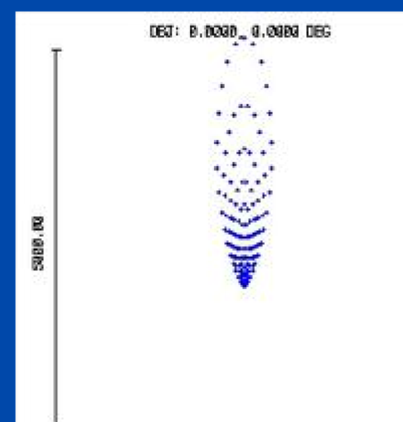
Spherical aberration as the mother of astigmatism





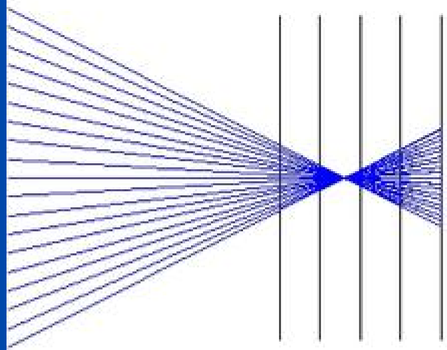
Big bundle of spherically aberrated rays



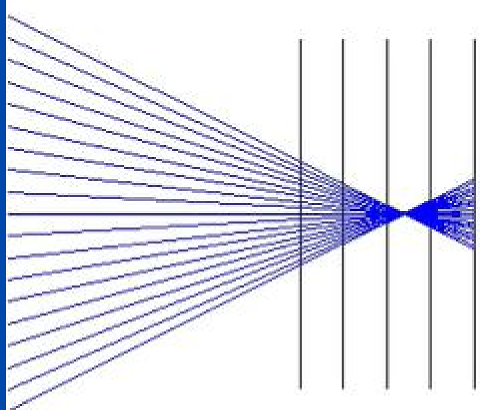


More-decentered subset of rays produces astigmatism

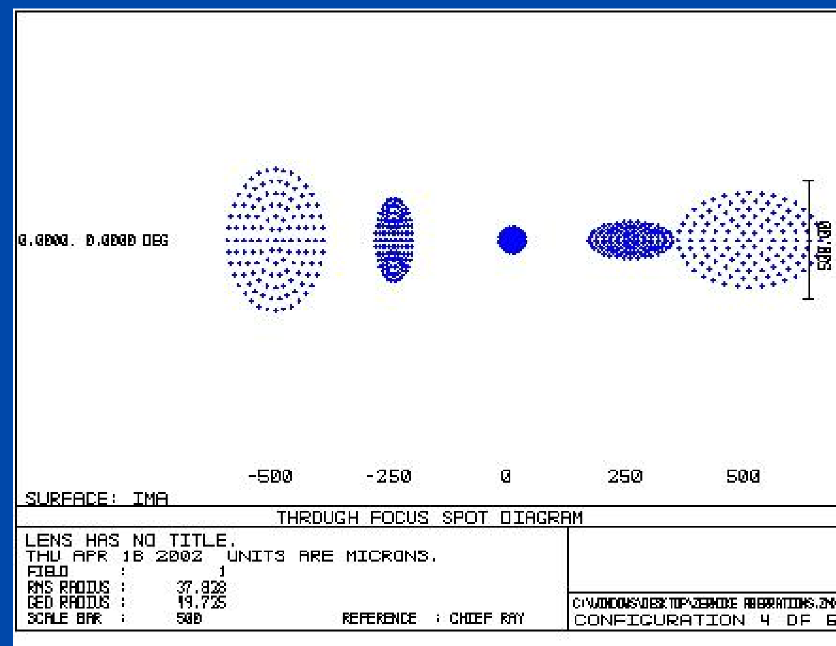
Astigmatism



Top view of rays



Side view of rays



Through-focus spot diagram
for astigmatism

Different view of astigmatism

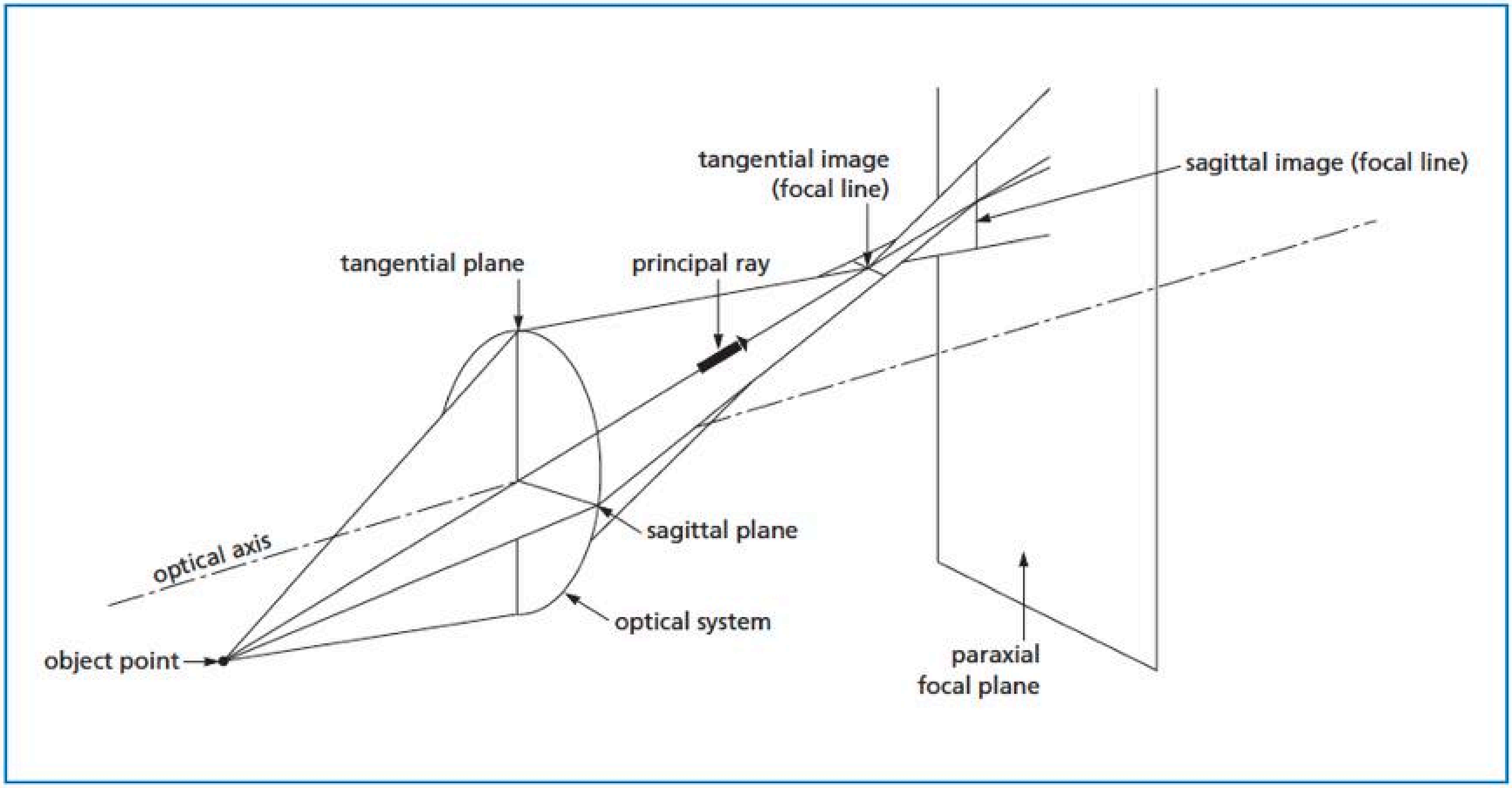
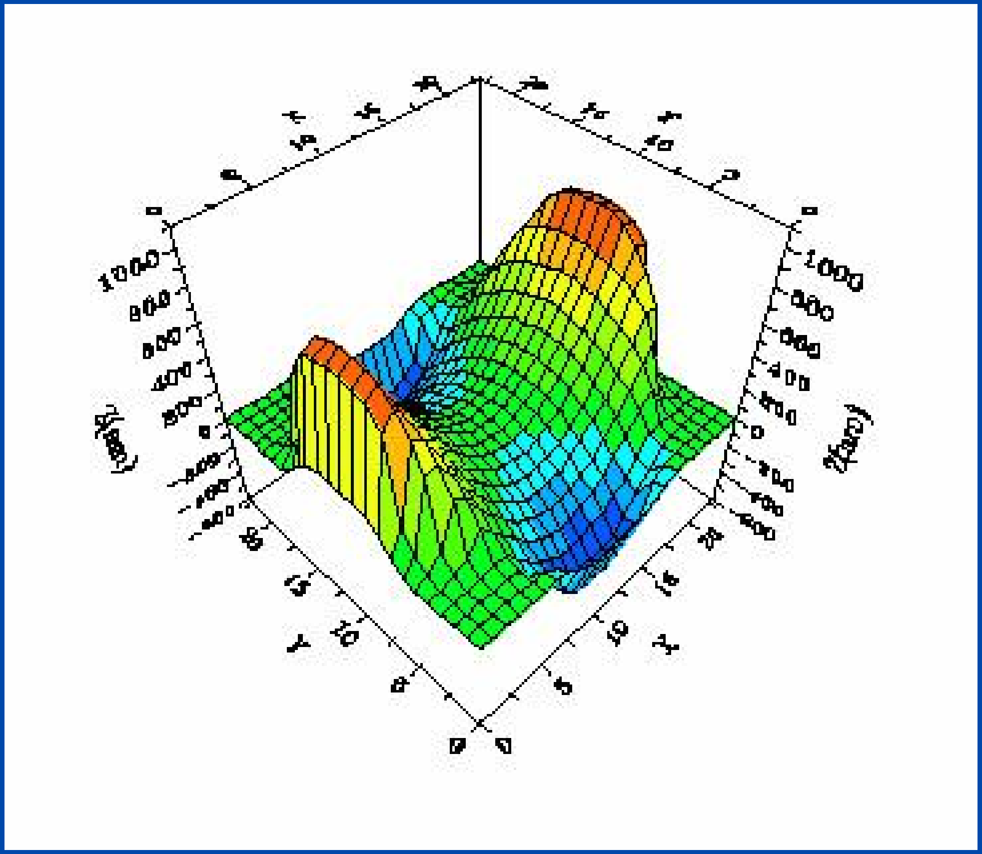


Figure 1.16 **Astigmatism represented by sectional views**

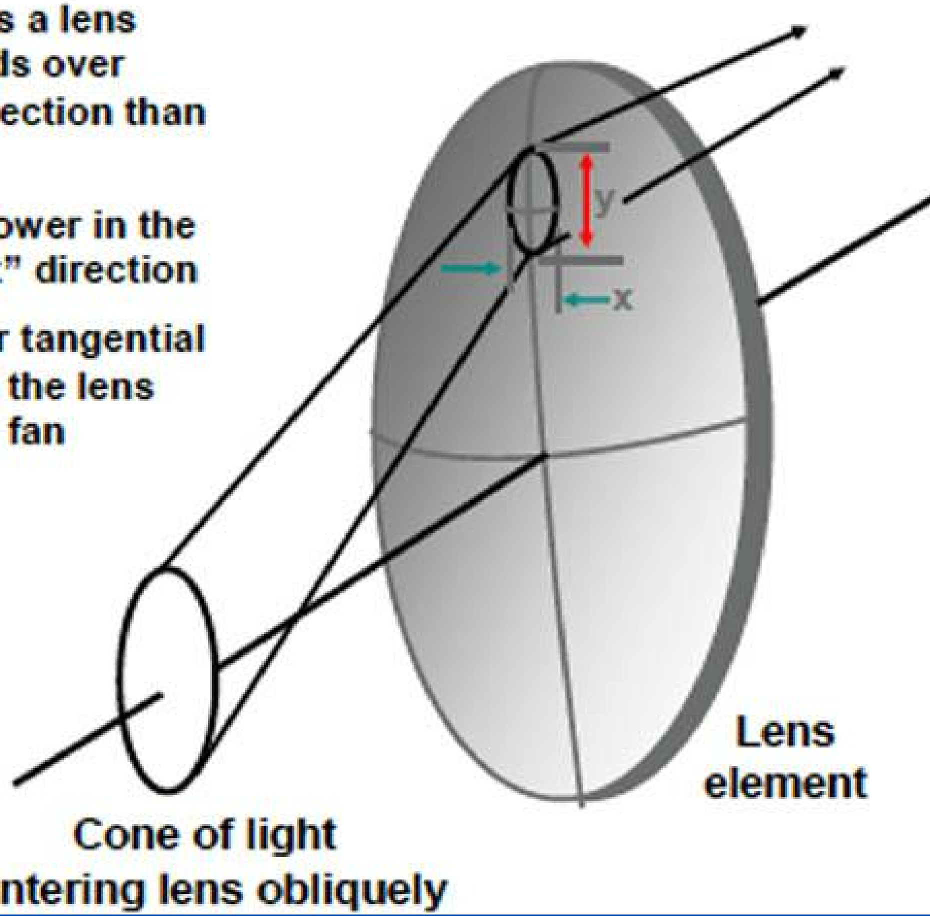
Credit: Melles-Griot

Wavefront for astigmatism



Where does astigmatism come from?

- When a cone of light enters a lens surface obliquely, it extends over more surface in the "y" direction than I the "x" direction
- This will introduce more power in the "y" direction than in the "x" direction
- The result is that the "y" or tangential ray fan will focus closer to the lens than the "x" or sagittal ray fan
- This is astigmatism

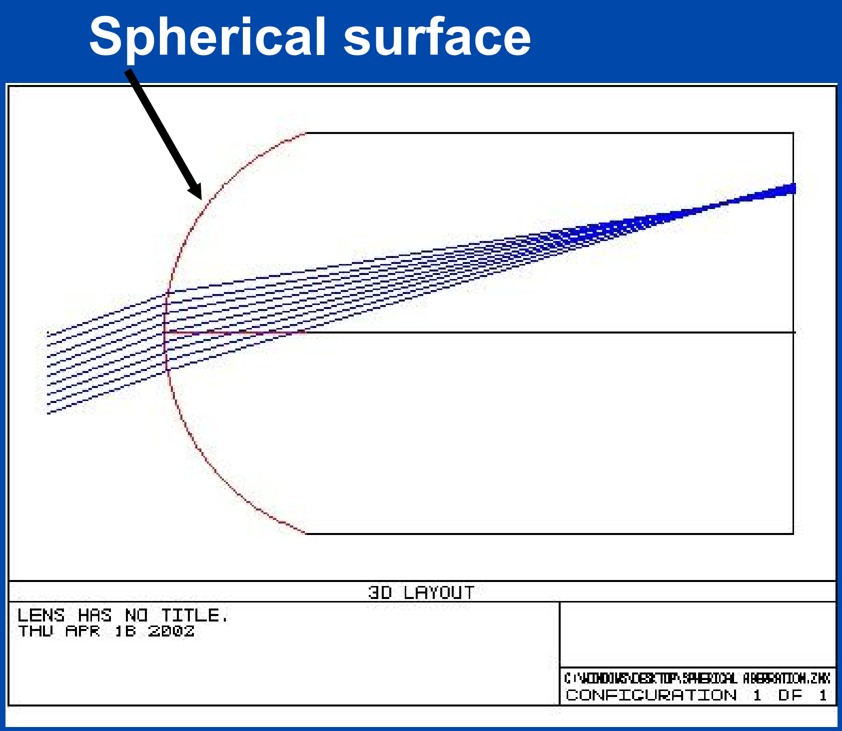


From Ian McLean, UCLA

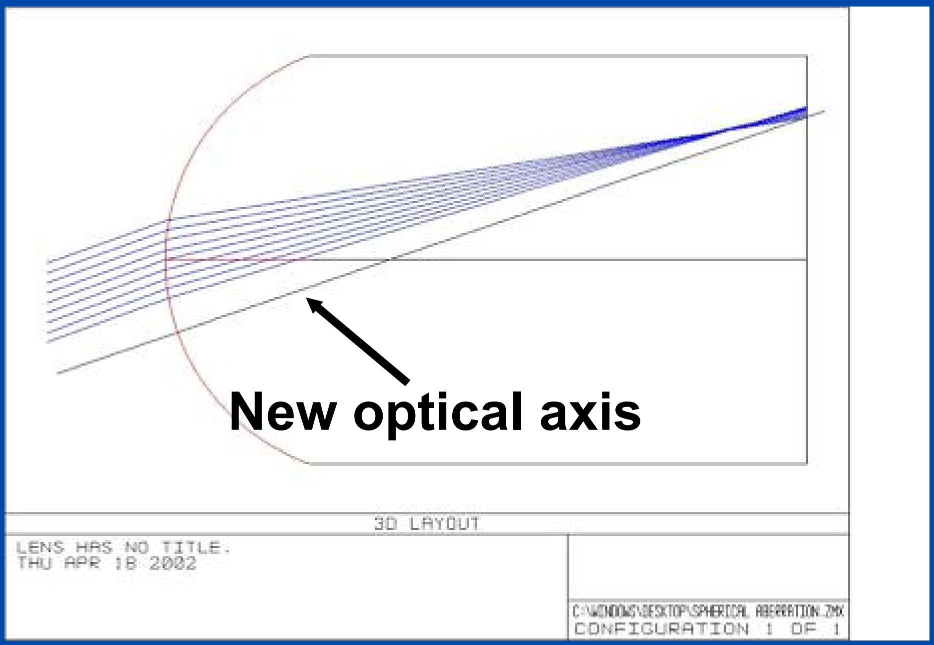
Concept Question

- How do you suppose eyeglasses correct for astigmatism?

Off-axis object is equivalent to having a de-centered ray bundle



Ray bundle from an off-axis object. How to view this as a de-centered ray bundle?



For any field angle there will be an optical axis, which is ⊥ to the surface of the optic and // to the incoming ray bundle. The bundle is de-centered wrt this axis.

Aberrations

- In optical systems
- Description in terms of Zernike polynomials
- Aberrations due to atmospheric turbulence

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Zernike Polynomials

- Convenient basis set for expressing wavefront aberrations over a *circular pupil*
- Zernike polynomials are orthogonal to each other
- A few different ways to normalize – always check definitions!

Expansion of the Phase in Zernike Polynomials

An alternative characterization of the phase comes from expanding φ in terms of a complete set of functions and then characterizing the coefficients of the expansion:

$$\varphi\,(r,\theta) = \Sigma\, a_{m,n} Z_{m,n}(r,\theta)$$

$$Z_{0,0} = 1 \qquad \text{piston}$$

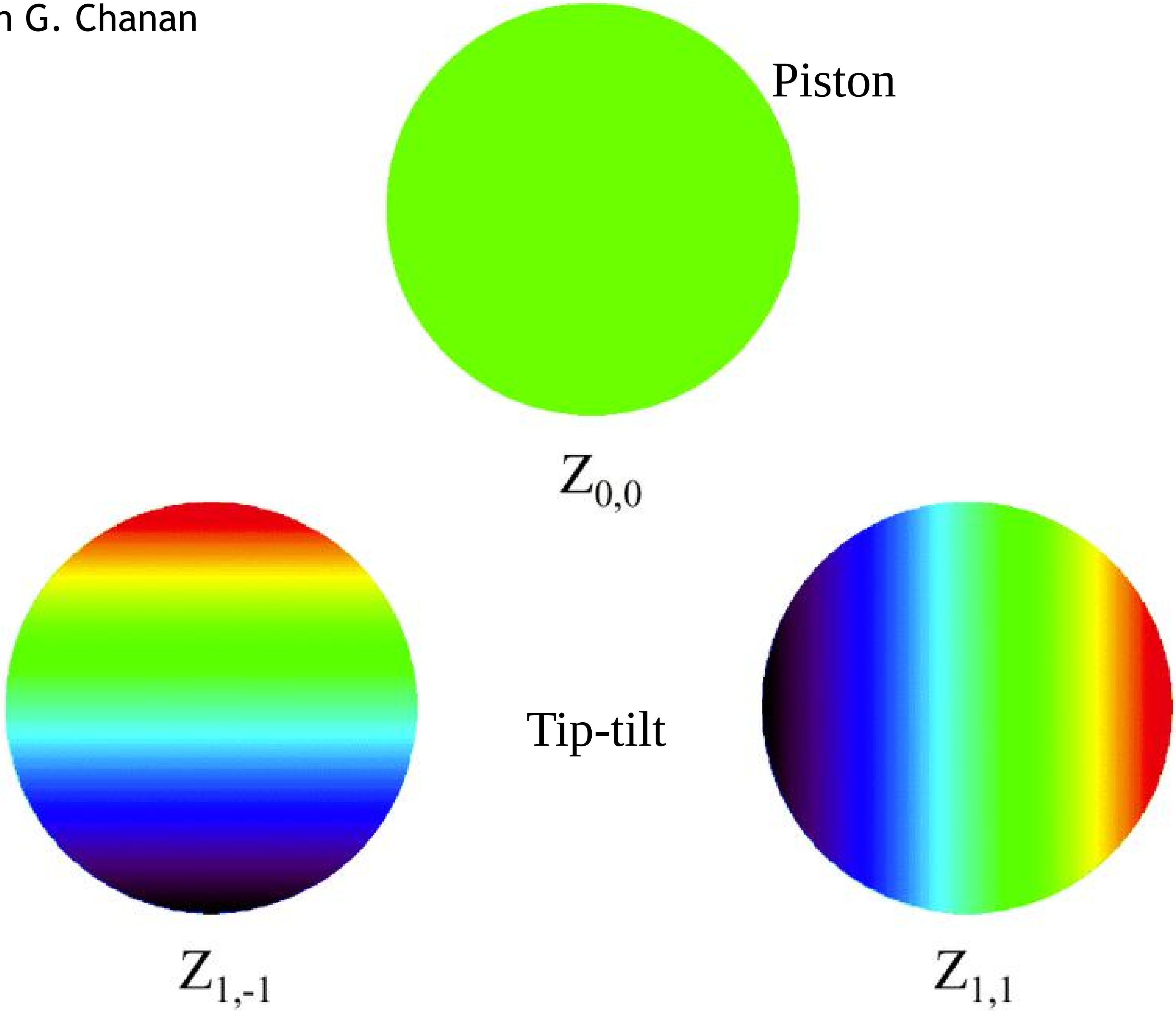
$$\left.\begin{aligned} Z_{1,-1} &= 2\,r\sin\theta \\ Z_{1,1} &= 2\,r\cos\theta \end{aligned}\right\} \qquad \text{tip/tilt}$$

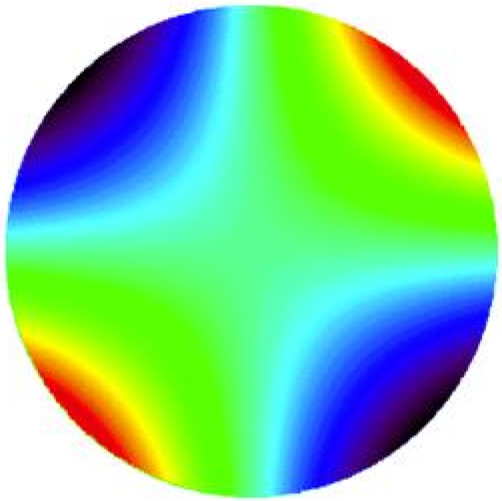
$$Z_{2,-2} = \sqrt{6}\, r^2 \sin 2\theta \qquad \text{astigmatism}$$

$$Z_{2,0} = \sqrt{3}\,(2r^2 - 1) \qquad \text{focus}$$

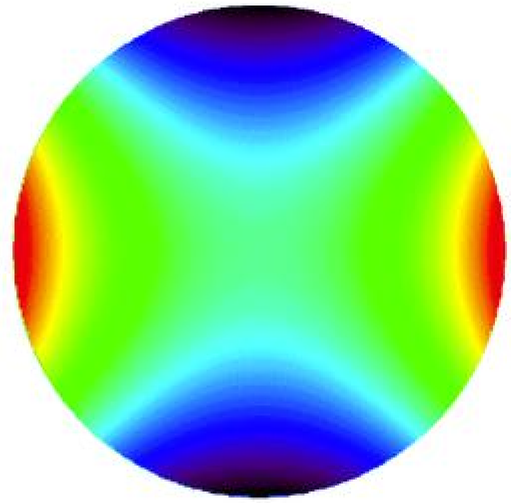
$$Z_{2,2} = \sqrt{6}\, r^2 \cos 2\theta \qquad \text{astigmatism}$$

From G. Chanan



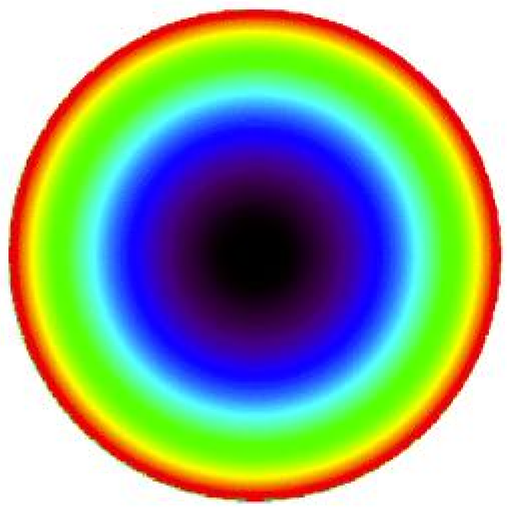


$Z_{2,-2}$



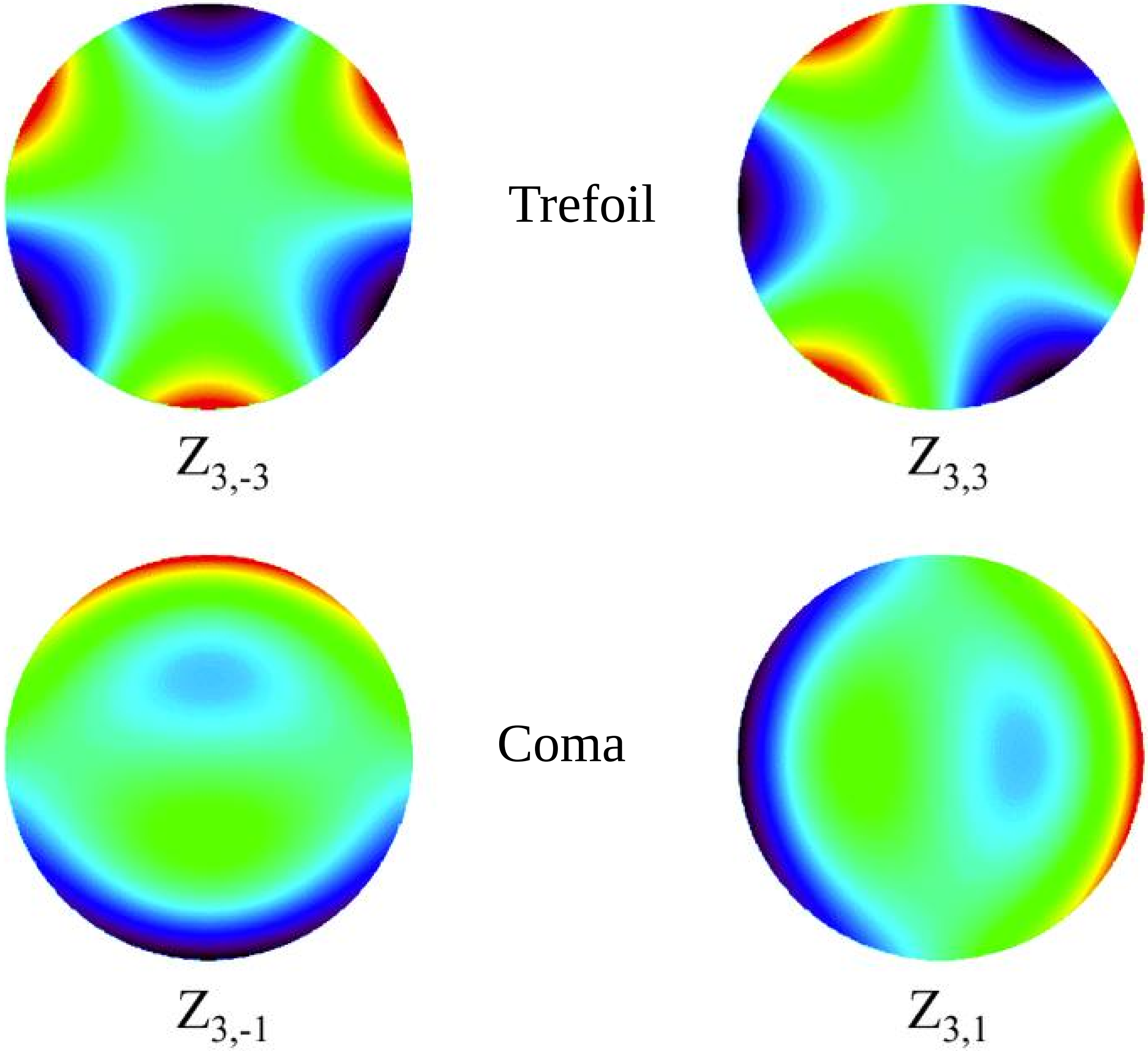
$Z_{2,2}$

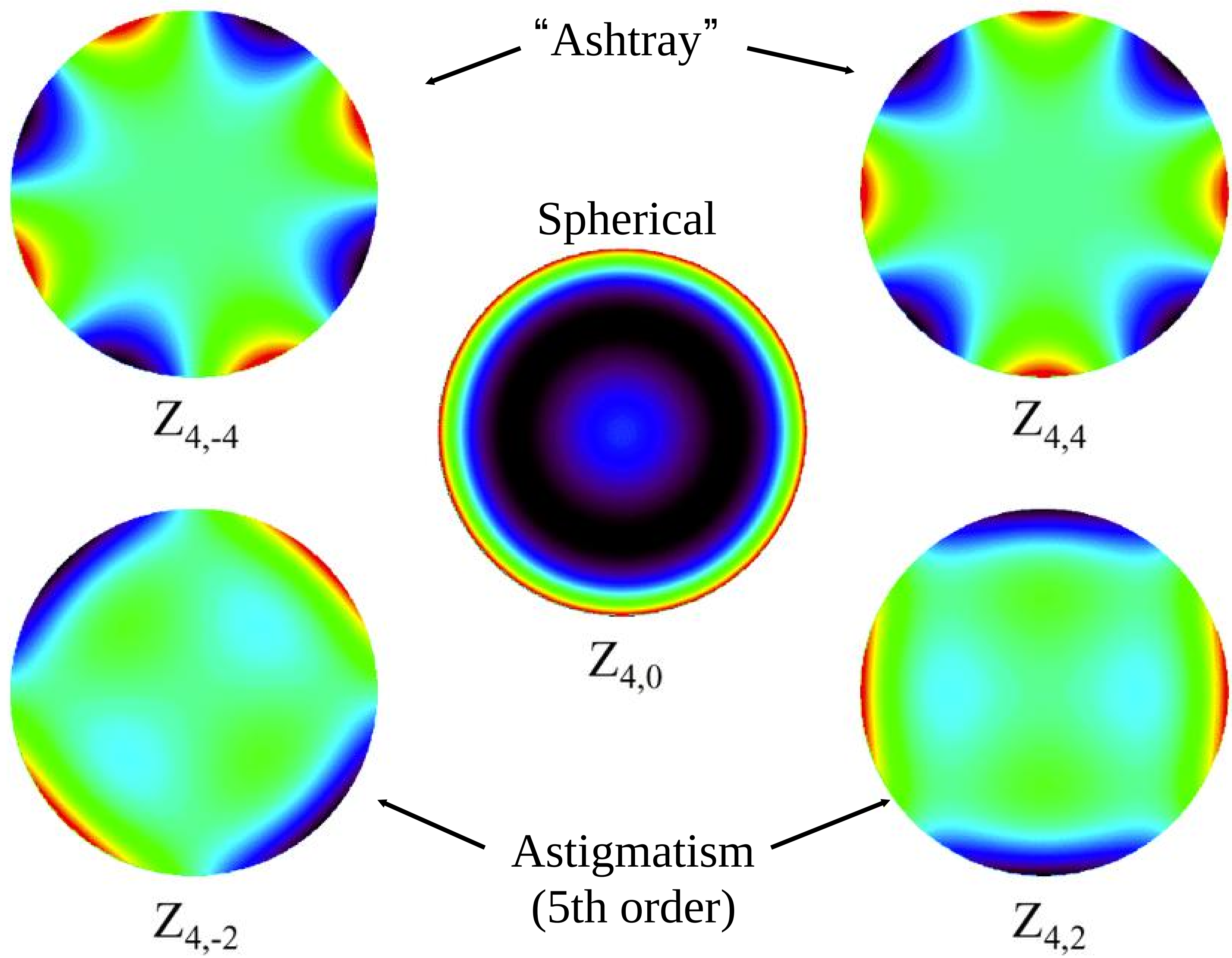
Astigmatism
(3rd order)

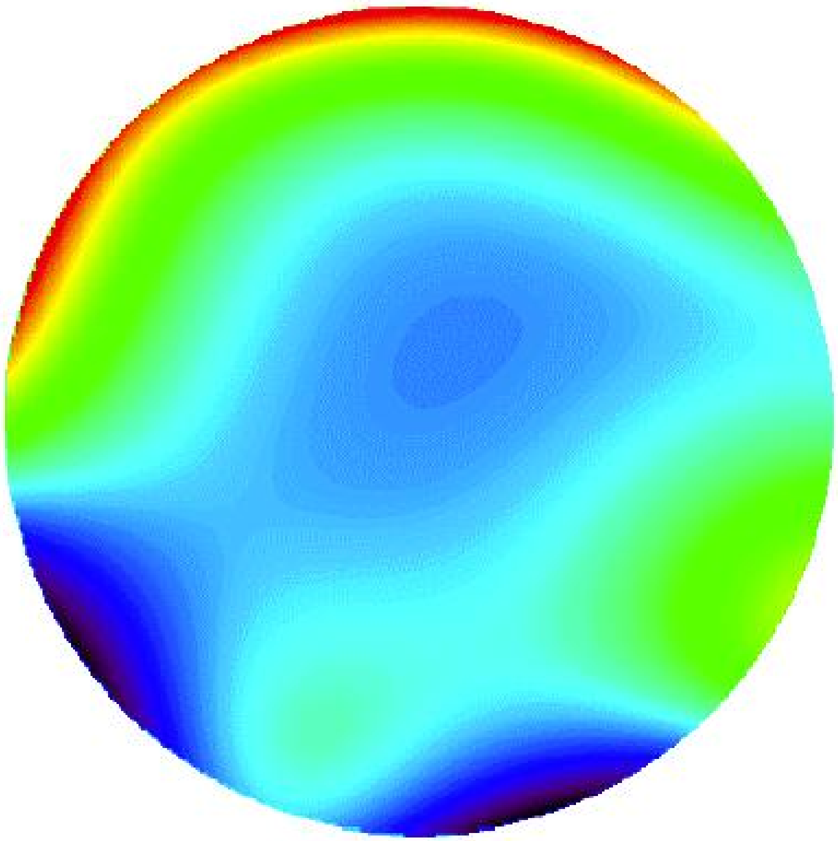


$Z_{2,0}$

Defocus


Trefoil
$Z_{3,-3}$
$Z_{3,3}$
Coma
$Z_{3,-1}$
$Z_{3,1}$


"Ashtray"
Spherical
$Z_{4,-4}$
$Z_{4,4}$
$Z_{4,0}$
$Z_{4,-2}$
$Z_{4,2}$
Astigmatism
(5th order)



Random Zernikes

Aberrations

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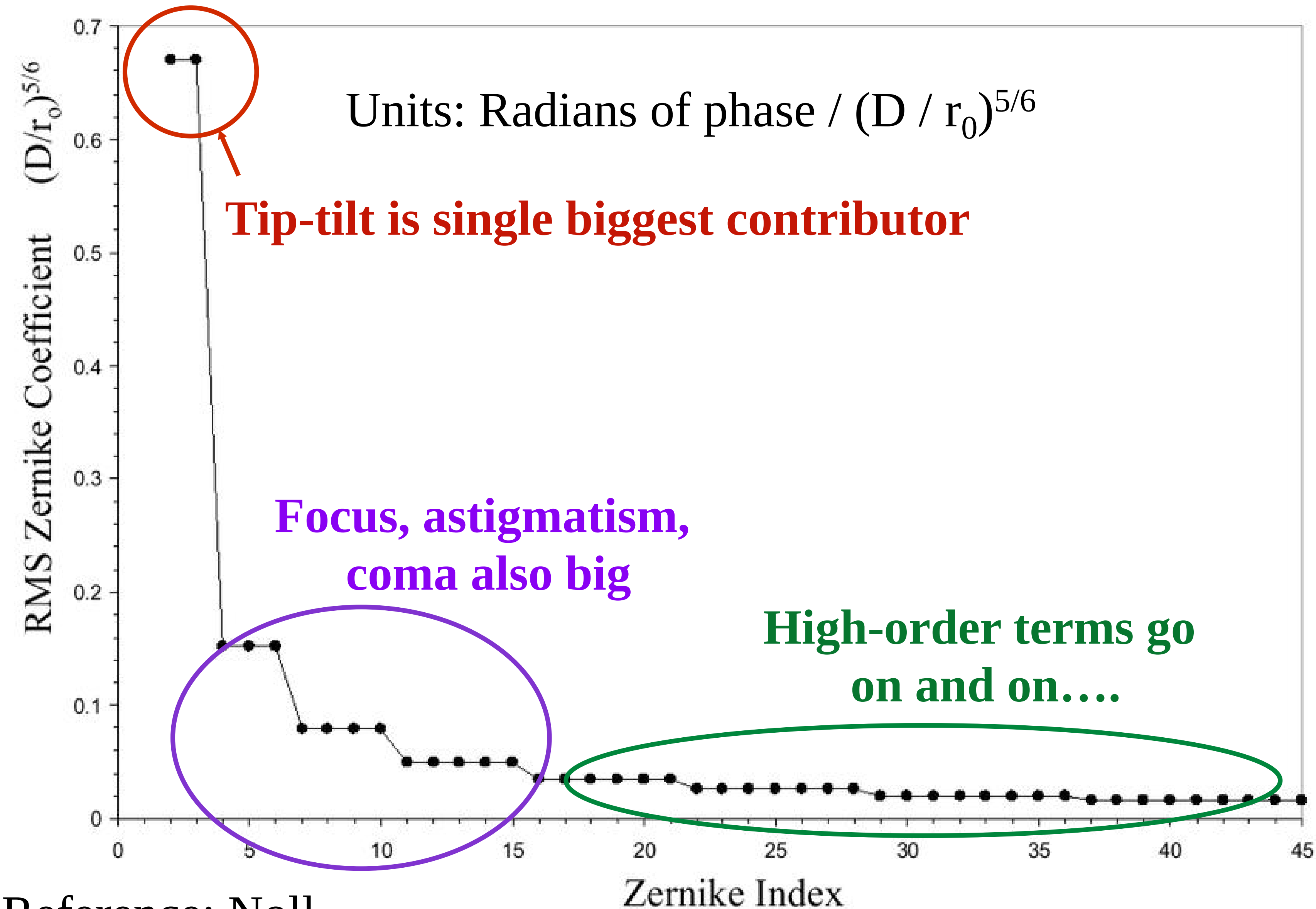
Atmospheric Zernike Coefficients



Reference: Noll

Seidel polynomials vs. Zernike polynomials

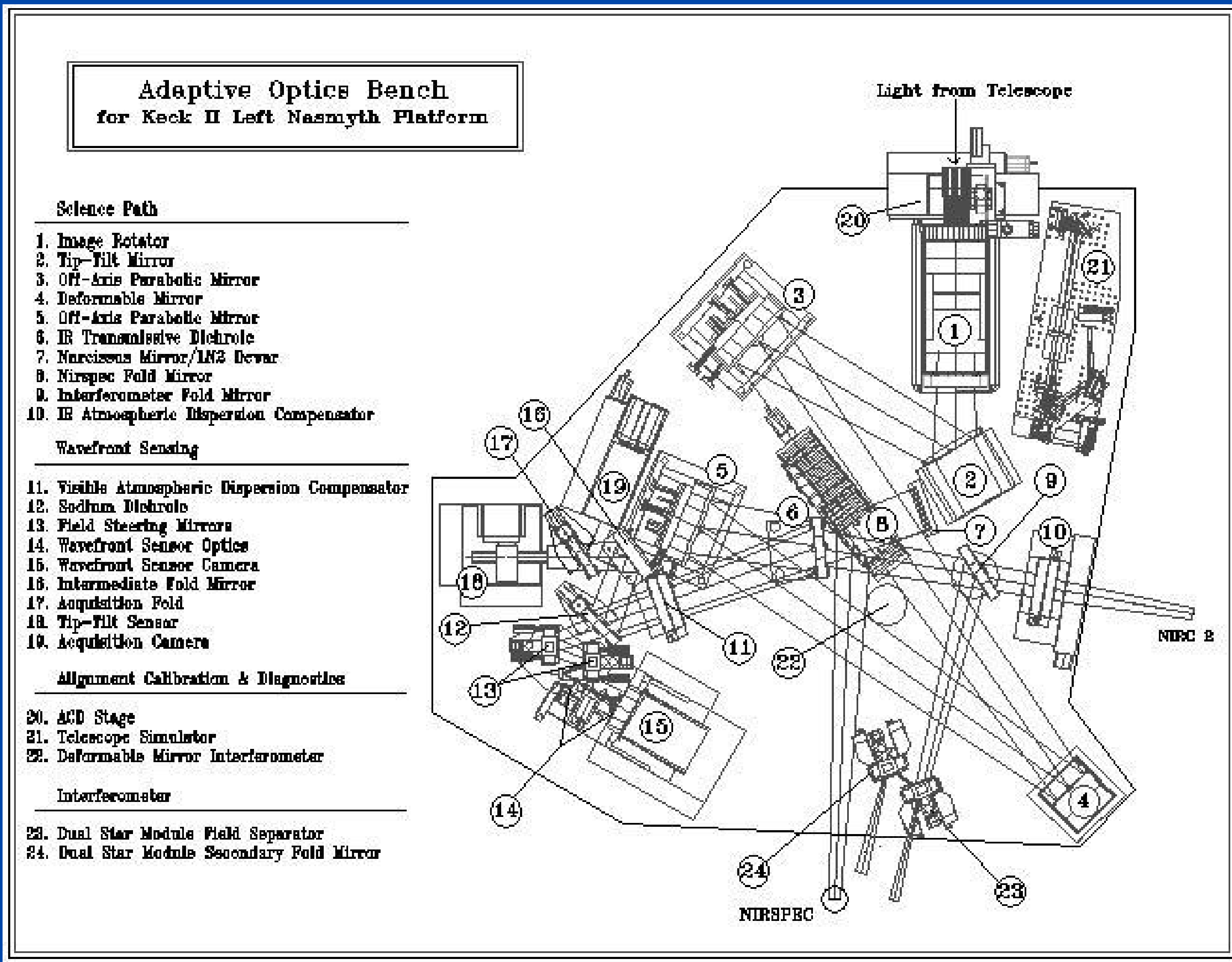
- **Seidel polynomials also describe aberrations**
- **At first glance, Seidel and Zernike aberrations look very similar**
- **Zernike aberrations are an orthogonal set of functions used to decompose a given wavefront *<u>at a given field point</u>* into its components**
  - Zernike modes add to the Seidel aberrations the correct amount of low-order modes to minimize rms wavefront error
- **Seidel aberrations are used in optical design to predict the aberrations in a design and how they will vary *<u>over the system's field of view</u>***
- **The Seidel aberrations have an analytic field-dependence that is proportional to some power of field angle**

References for Zernike Polynomials

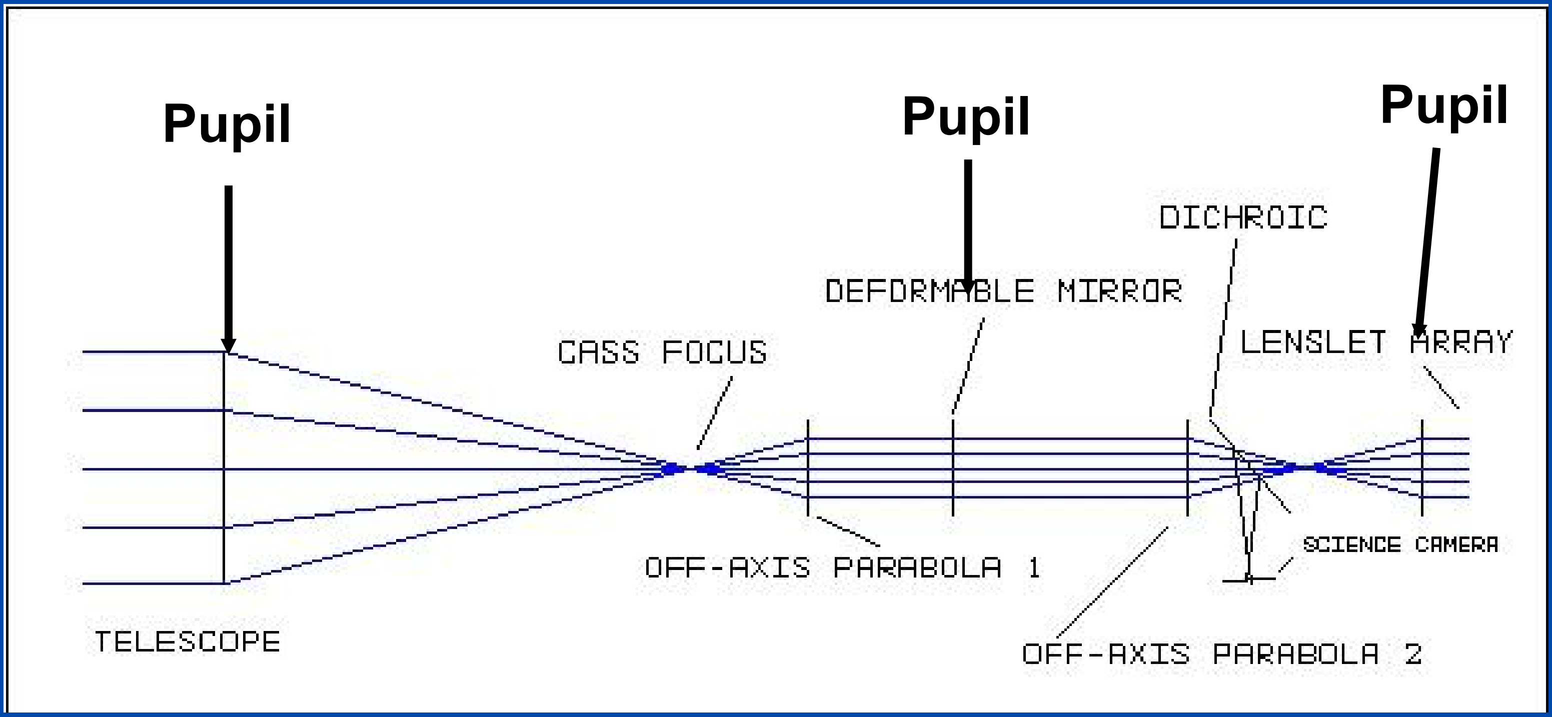
- **Pivotal Paper: Noll, R. J. 1976, “Zernike polynomials and atmospheric turbulence”, JOSA 66, page 207**

- **Books:**
  - e.g. Hardy, Adaptive Optics, pages 95-96

Let's get back to design of AO systems
Why on earth does it look like this ??



Considerations in the optical design of AO systems: pupil relays

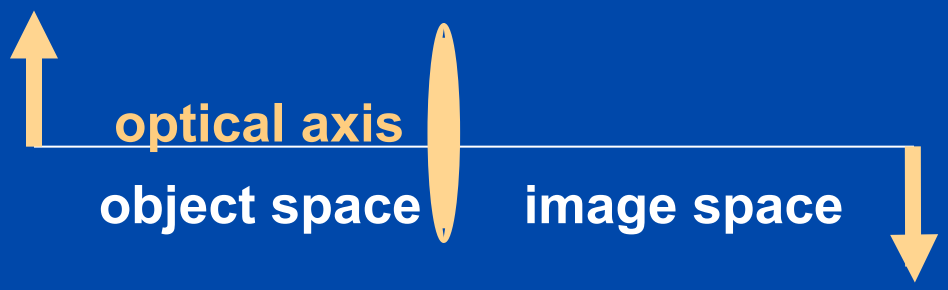


Deformable mirror and Shack-Hartmann lenslet array should be "optically conjugate to the telescope pupil."

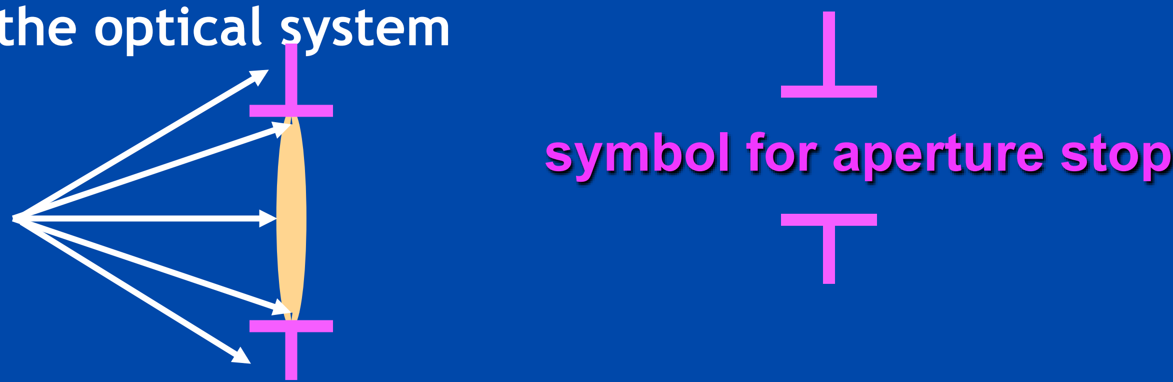
What does this mean?

Define some terms

- “Optically conjugate” = “image of....”



- “Aperture stop” = the aperture that limits the bundle of rays accepted by the optical system



So now we can translate:

- “The deformable mirror should be optically conjugate to the telescope pupil”

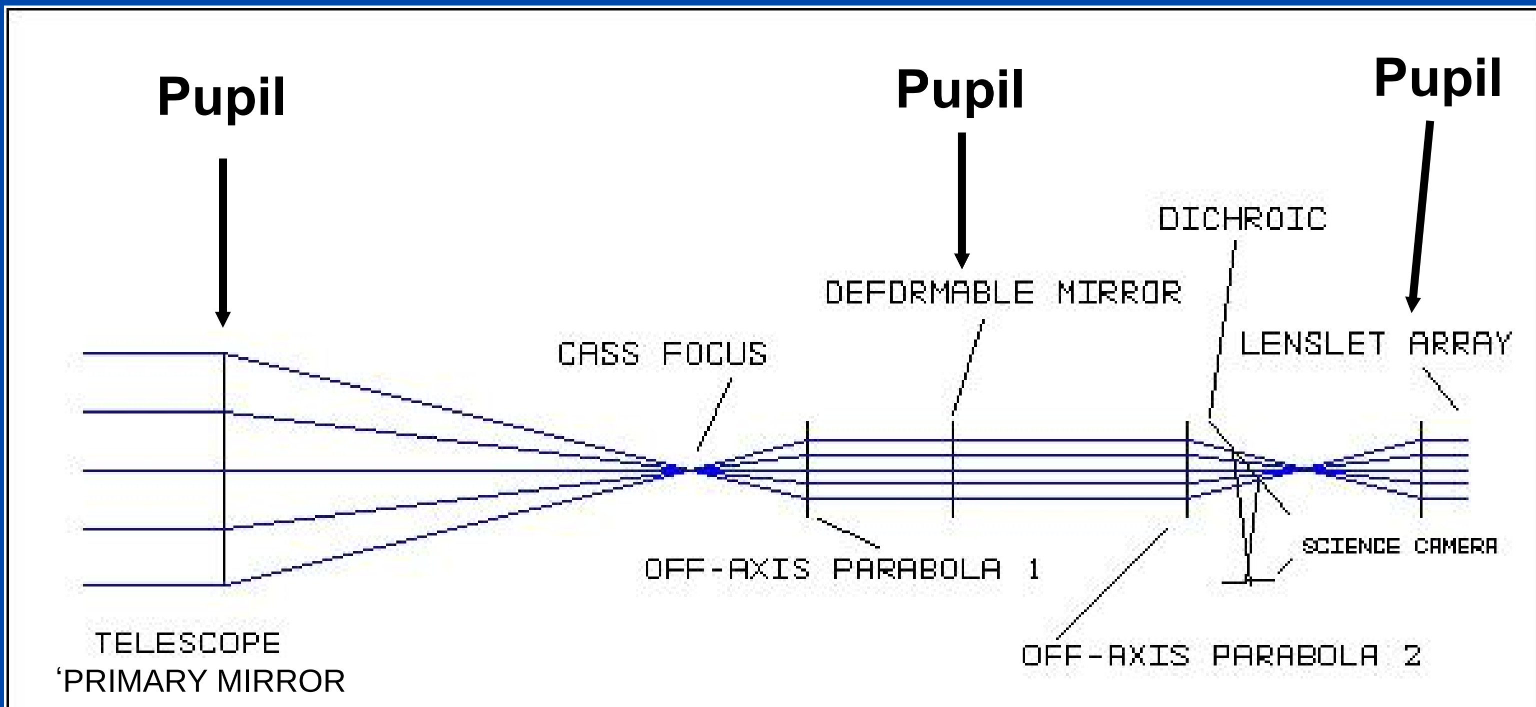
  means

- The surface of the deformable mirror is an image of the telescope pupil

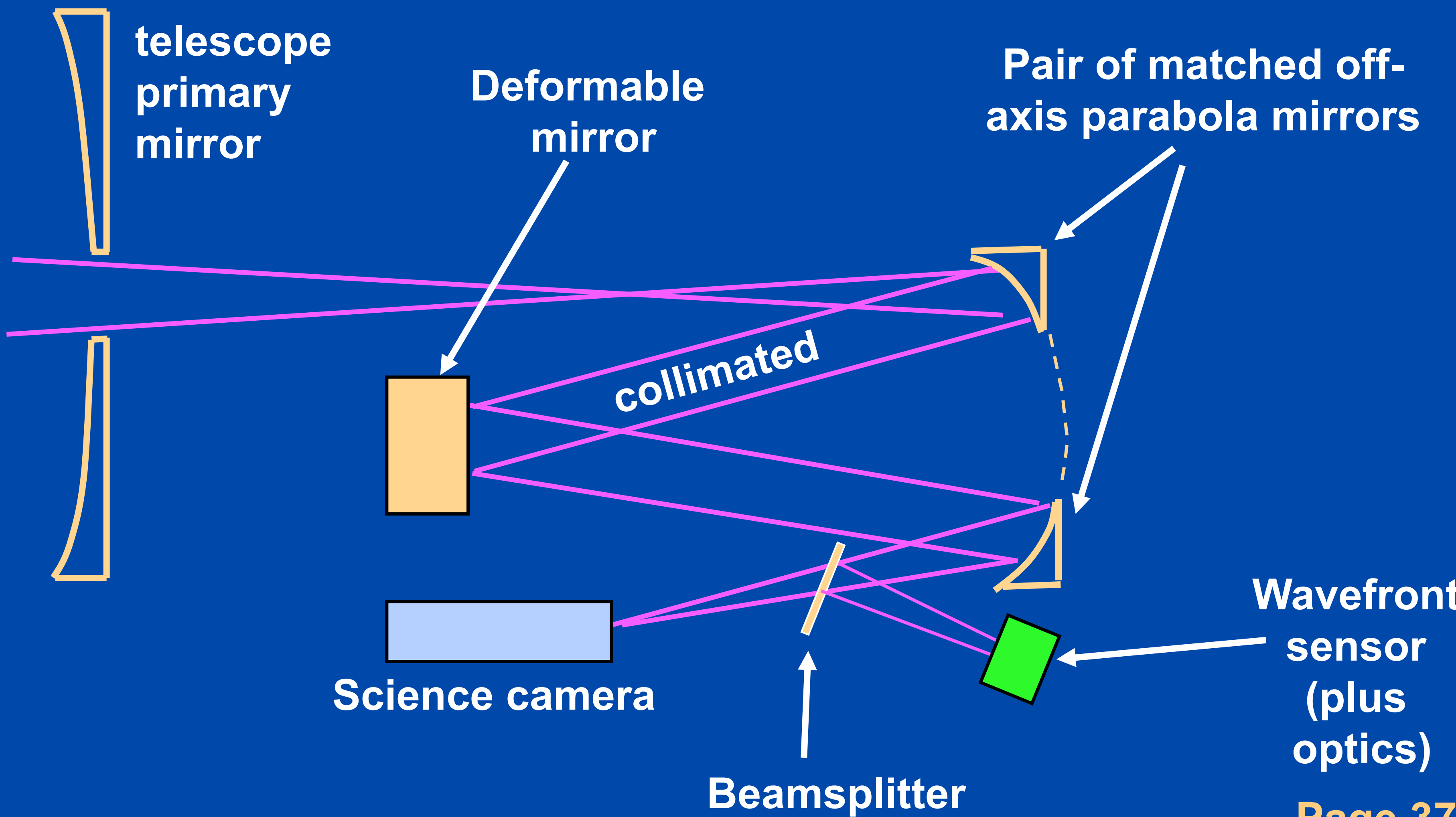
  where

- The pupil is an image of the aperture stop
  - In practice, the pupil is usually the primary mirror of the telescope

Considerations in the optical design of AO systems: "pupil relays"

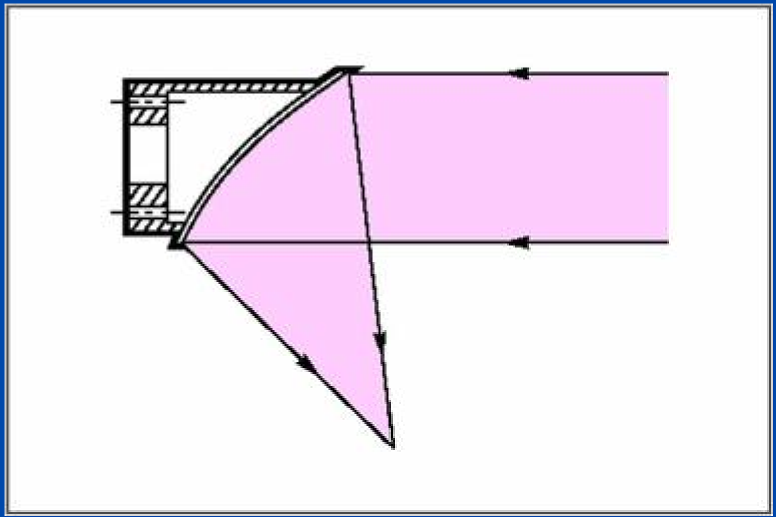


Typical optical design of AO system



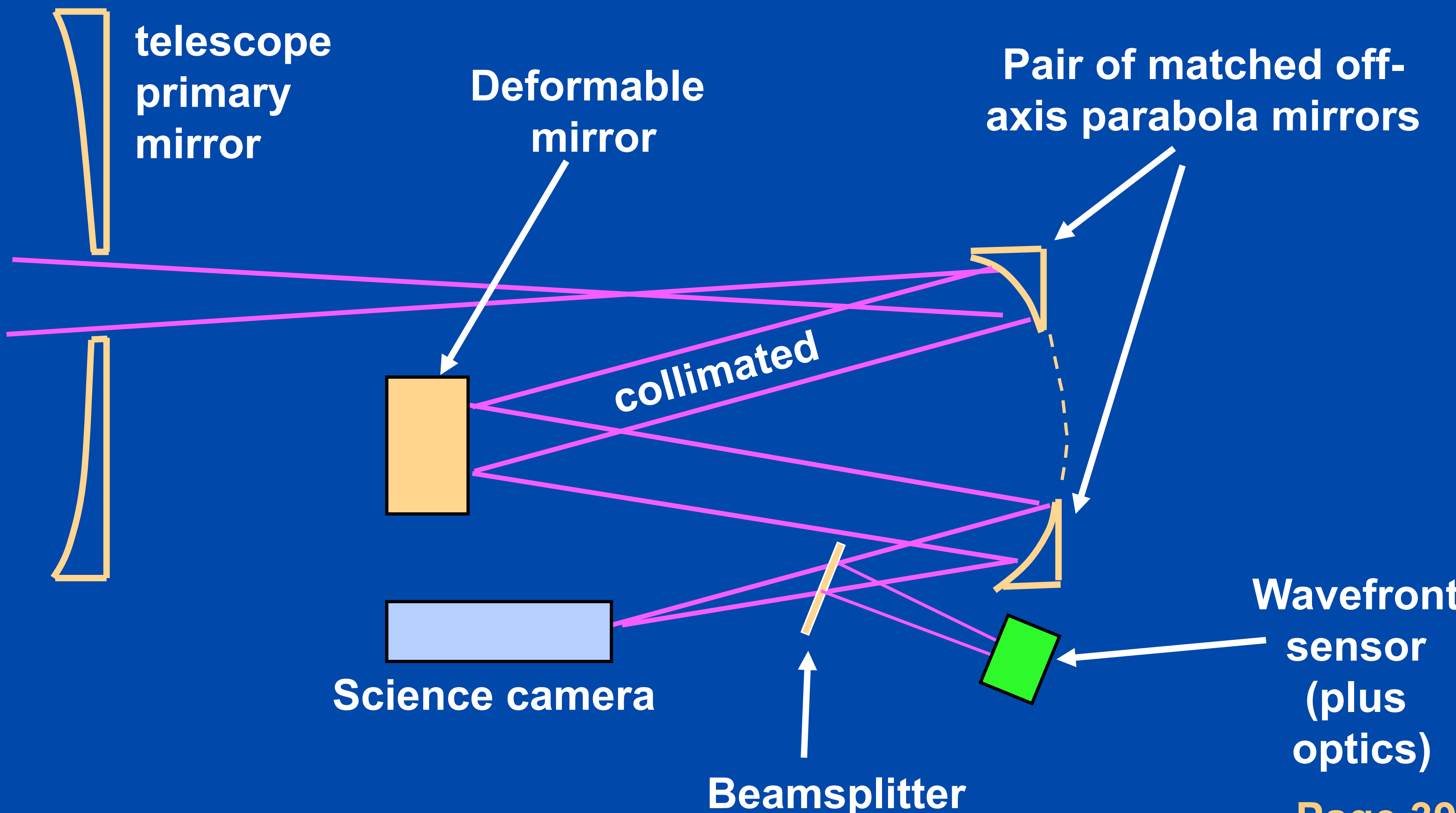
More about off-axis parabolas

- Circular cut-out of a parabola, off optical axis
- Frequently used in matched pairs (each cancels out the off-axis aberrations of the other) to first collimate light and then refocus it



SORL

Concept Question: what elementary optical calculations would you have to do, to lay out this AO system? (Assume you know telescope parameters, DM size)

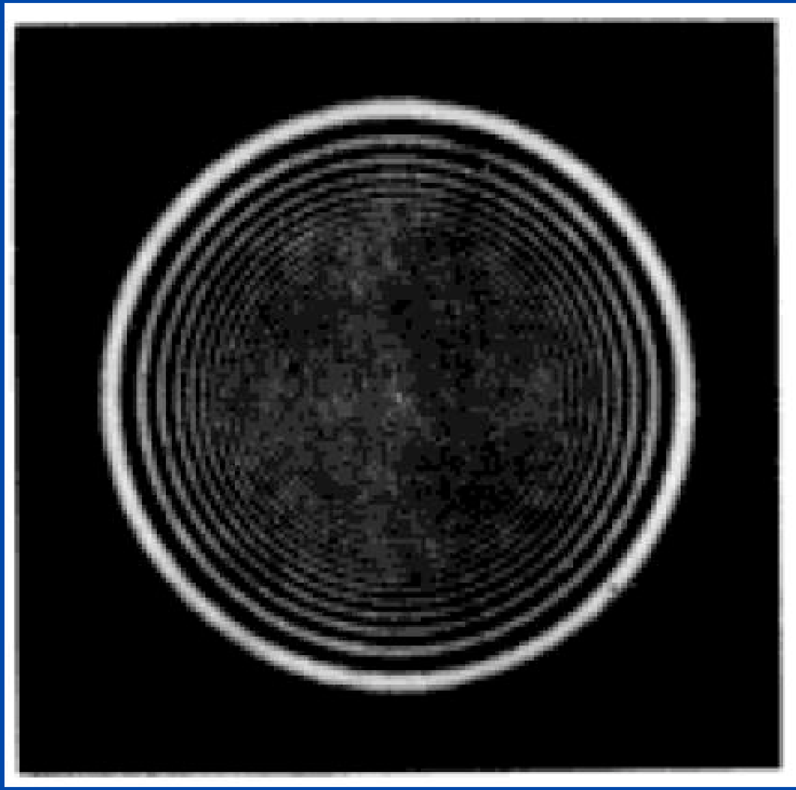


Review of important points

- **Both lenses and mirrors can focus and collimate light**
- **Equations for system focal lengths, magnifications are quite similar for lenses and for mirrors**
- **Telescopes are combinations of two or more optical elements**
  - Main function: to gather lots of light
- **Aberrations occur both due to your local instrument’s optics and to the atmosphere**
  - Can describe both with Zernike polynomials
- **Location of pupils is important to AO system design**

Part 2: Fourier (or Physical) Optics

Wave description: diffraction, interference



Diffraction of light by a circular aperture

Levels of models in optics

Geometric optics - rays, reflection, refraction

↓

Physical optics (Fourier optics) - diffraction, scalar waves

↓

Electromagnetics - vector waves, polarization

↓

Quantum optics - photons, interaction with matter, lasers

Maxwell's Equations: Light as an electromagnetic wave (Vectors!)

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

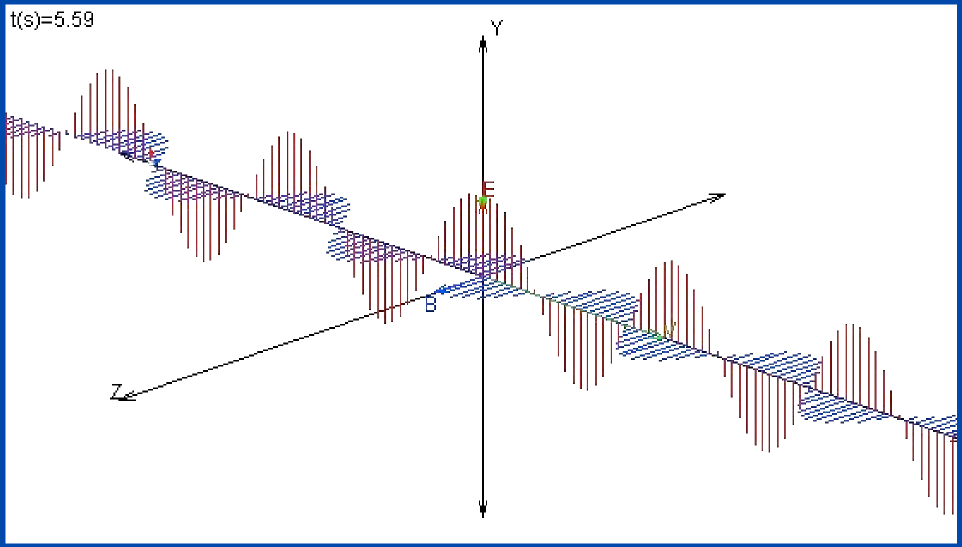
$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times B = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c}\mathbf{J}$$

Light as an EM wave

- Light is an electromagnetic wave phenomenon, **E** and **B** are perpendicular
- We detect its presence because the EM field interacts with matter (pigments in our eye, electrons in a CCD, …)



Physical Optics is based upon the scalar Helmholtz Equation (no polarization)

- In free space

$$\nabla^2 E_\perp = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_\perp$$

- Traveling waves

$$E_\perp(x,t) = E_\perp(0, t \pm x/c)$$

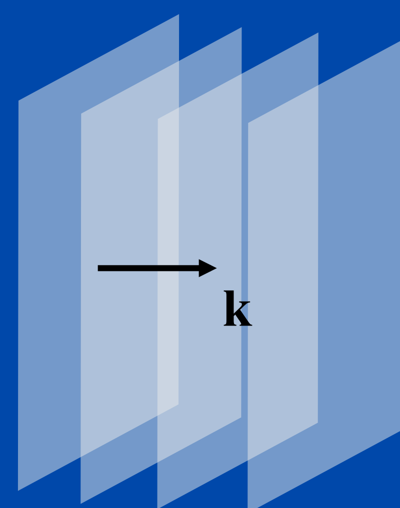
- Plane waves

$$E_\perp(\mathbf{x},t) = \tilde{E}(\mathbf{k})\, e^{i\,(\omega\, t - \mathbf{k}\cdot\mathbf{x})}$$

Helmholtz Eqn.,
Fourier domain

$$k^2 \tilde{E} = (\omega / c)^2\, \tilde{E}$$

$$k = \omega / c$$



Dispersion and phase velocity

- **In free space** $k = \omega / c$ where $k \equiv 2\pi / \lambda$ and $\omega \equiv 2\pi \nu$
  - Dispersion relation $k(\omega)$ is linear function of ω
  - Phase velocity or propagation speed = $\omega / k = c = const.$
- **In a medium**
  - Plane waves have a *phase velocity*, and hence a wavelength, that depends on frequency

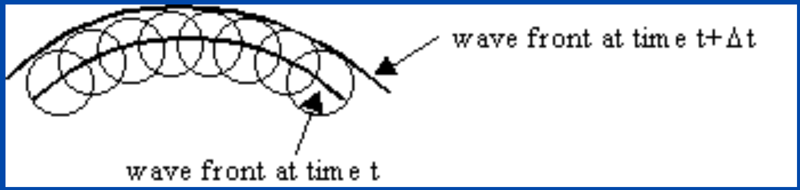
$$k(\omega) = \omega / \mathrm{v}_{phase}$$

  - The "slow down" factor relative to c is the *index of refraction*, $n(\omega)$

$$\mathrm{v}_{phase} = c / n(\omega)$$

Optical path – Fermat's principle

- Huygens' wavelets



- Optical distance to radiator:

$$\Delta x = \mathrm{v}\ \Delta t = \mathrm{c}\ \Delta t/n$$

$$\mathrm{c}\ \Delta t = n\ \Delta x$$

$$\text{Optical Path Difference} = \text{OPD} = \int n\,\mathrm{d}x$$

- Wavefronts are iso-OPD surfaces
- Light ray paths are paths of least* time (least* OPD)

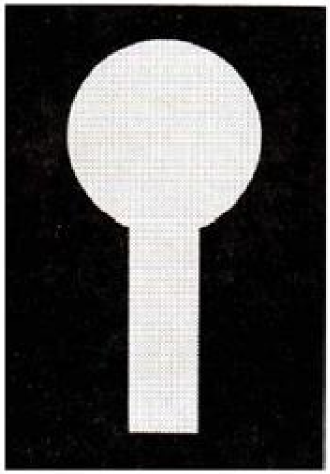
*in a local minimum sense

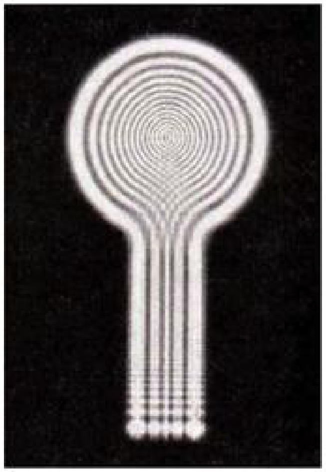
What is Diffraction?

When an opaque body is placed midway between an observing screen and a point source, diffraction effects produce an intricate shadow made up of bright and dark regions quite unlike anything one might expect from the principles of geometrical optics.

The phenomenon of *diffraction* has thus been defined as "any deviation of light rays from rectilinear paths that cannot be interpreted as reflection or refraction".

Aperture



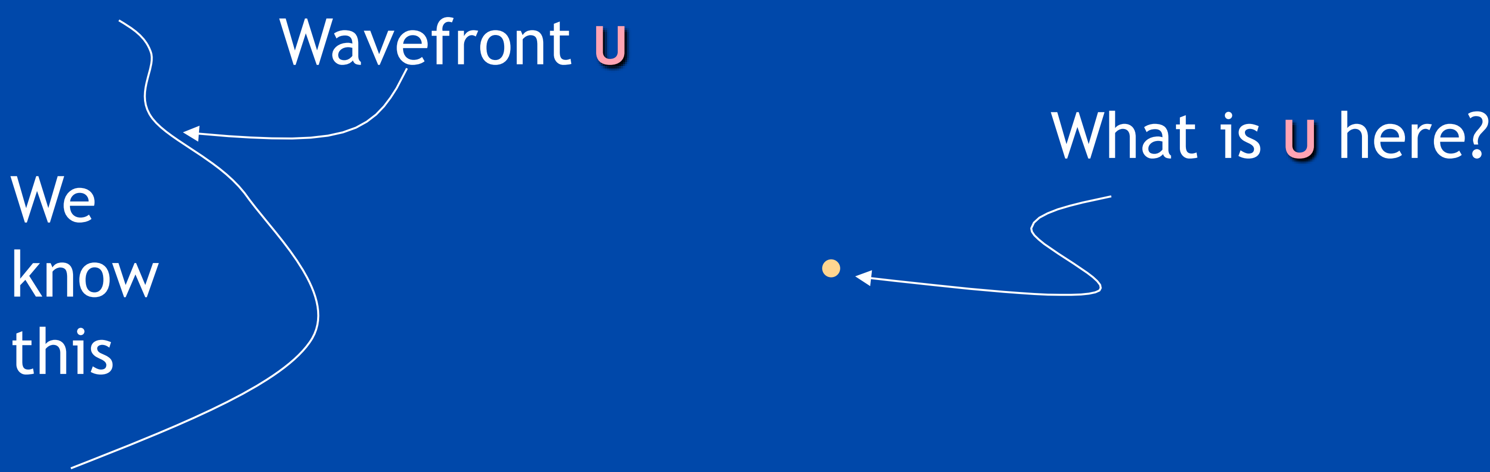


Light that has passed thru aperture, seen on screen downstream

In diffraction, apertures of an optical system limit the spatial extent of the wavefront

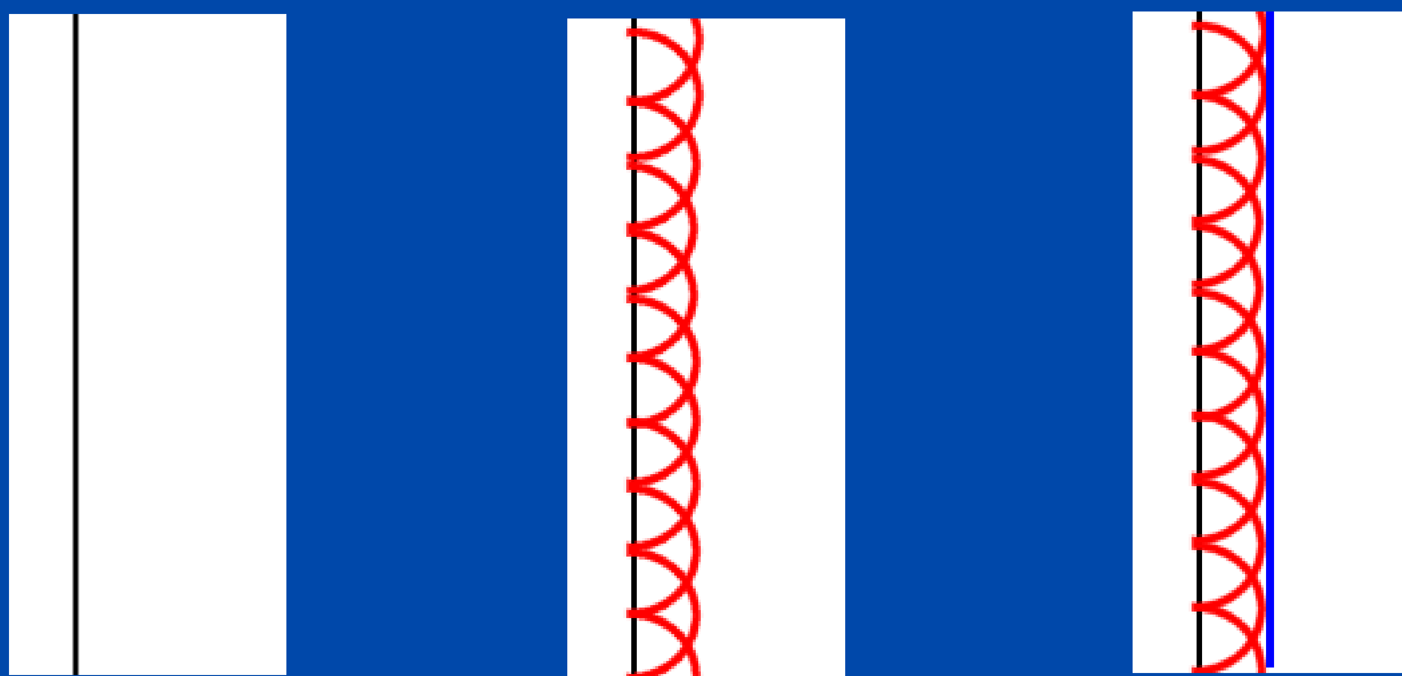
Credit: James E. Harvey, Univ. Central Florida

Diffraction Theory



Diffraction as one consequence of Huygens' Wavelets: Part 1

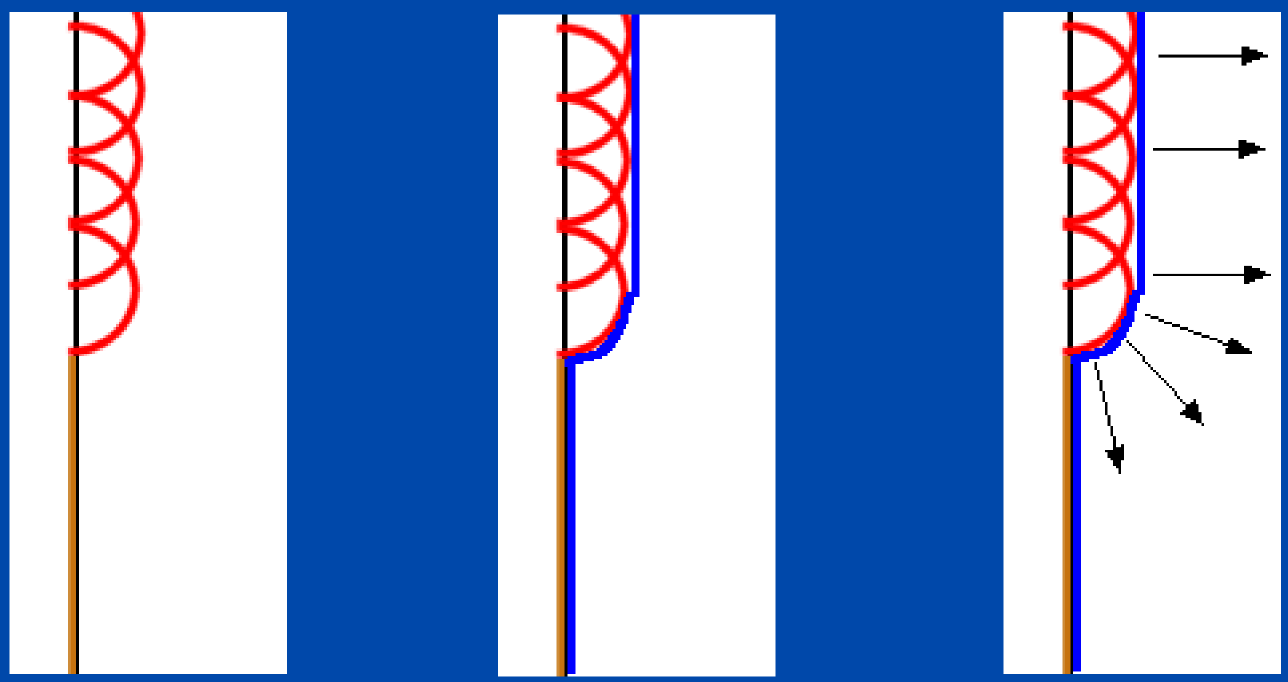
Every point on a wave front acts as a source of tiny wavelets that move forward.



Huygens' wavelets for an infinite plane wave

Diffraction as one consequence of Huygens' Wavelets: Part 2

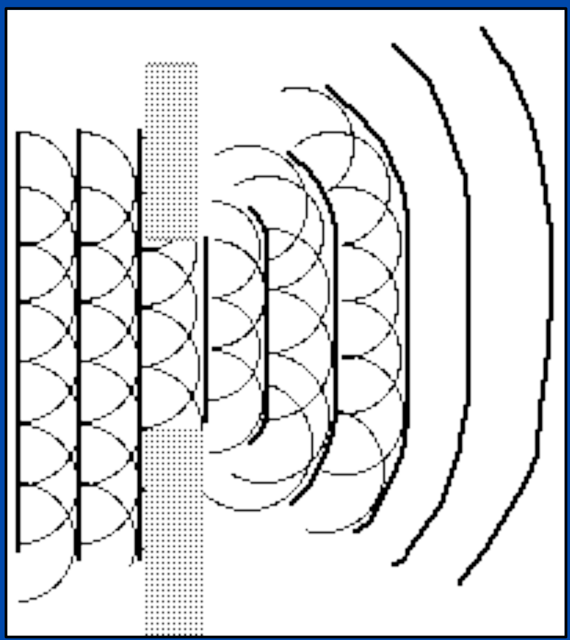
Every point on a wave front acts as a source of tiny wavelets that move forward.



Huygens' wavelets when part of a plane wave is blocked

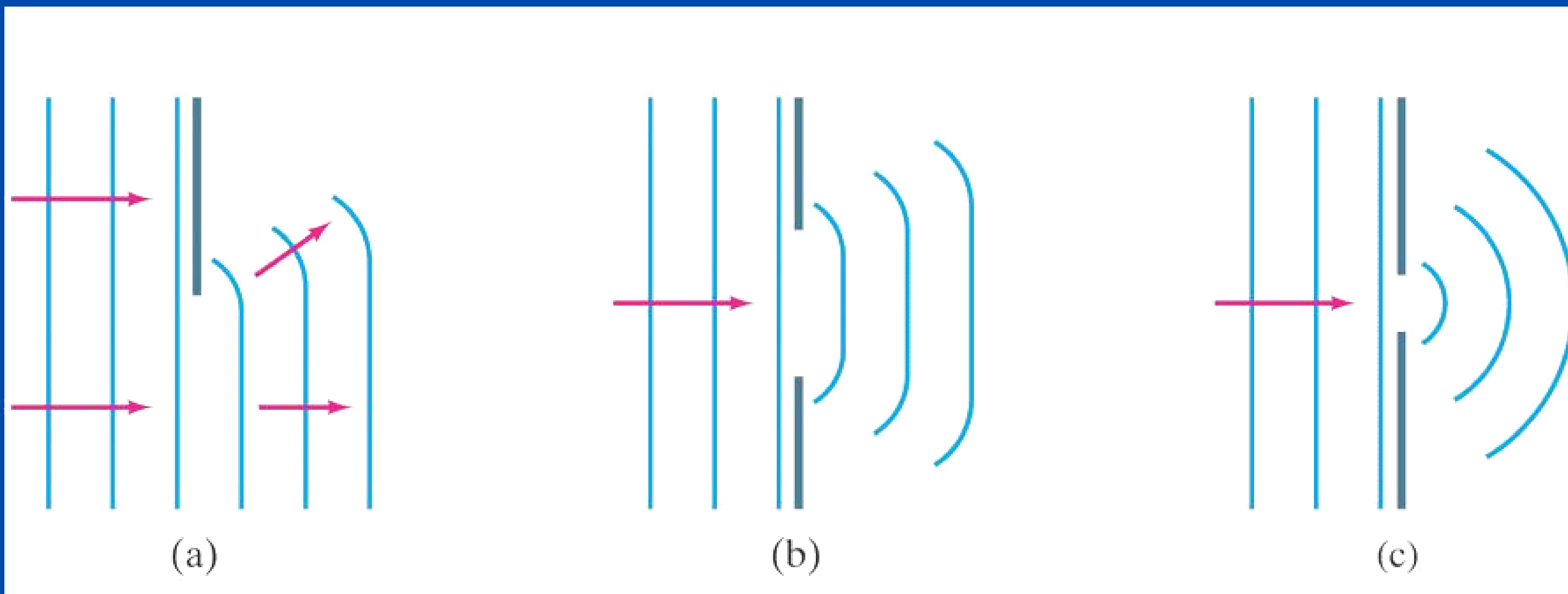
Diffraction as one consequence of Huygens' Wavelets: Part 3

Every point on a wave front acts as a source of tiny wavelets that move forward.



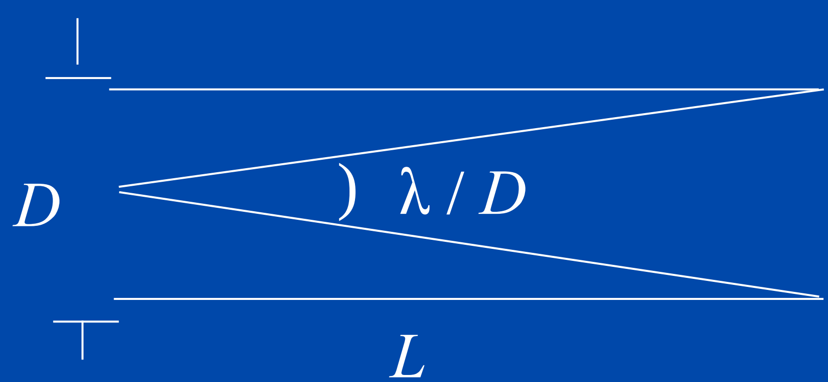
Huygens' wavelets for a slit

The size of the slit (relative to a wavelenth) matters



Rayleigh range

- Distance where diffraction overcomes paraxial beam propagation

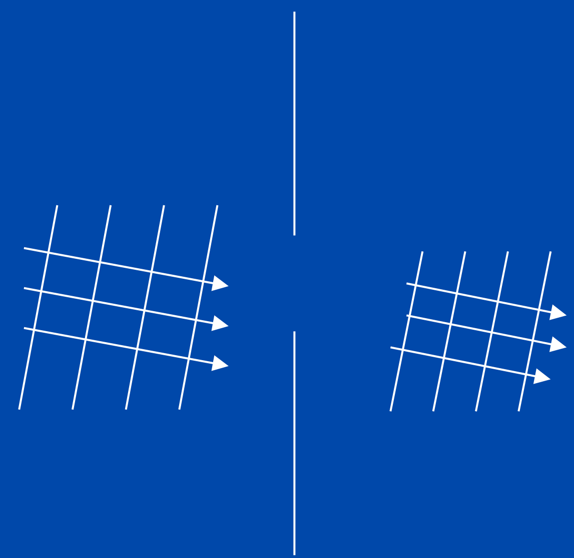


$$\frac{L\lambda}{D} = D \Rightarrow L = \frac{D^2}{\lambda}$$

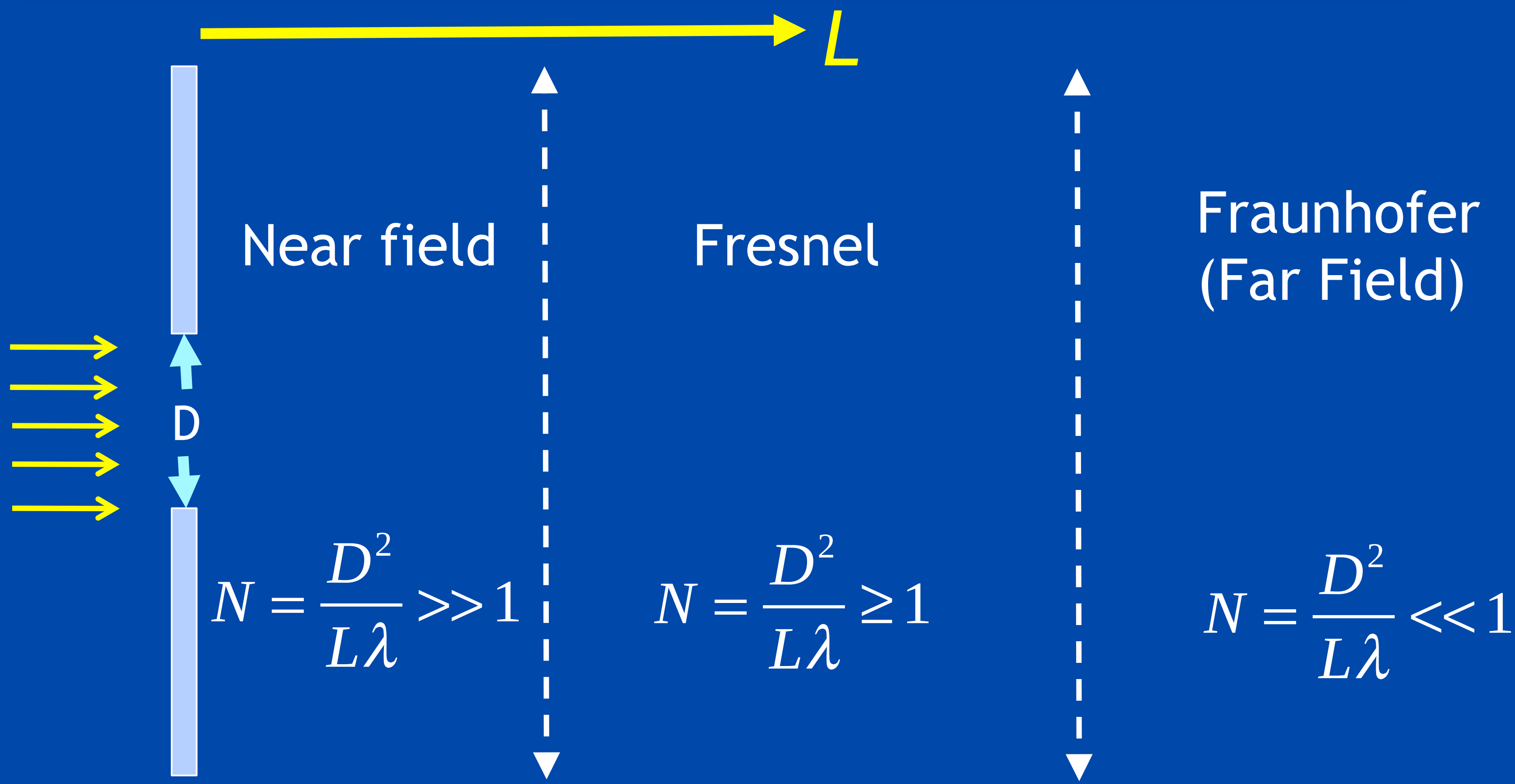
Fresnel vs. Fraunhofer diffraction

- Fresnel regime is the near-field regime: the wave fronts are curved, and their mathematical description is more involved.

- Very far from a point source, wavefronts almost plane waves.
- Fraunhofer approximation valid when source, aperture, and detector are all very far apart (or when lenses are used to convert spherical waves into plane waves)



Regions of validity for diffraction calculations



The farther you are from the slit, the easier it is to calculate the diffraction pattern

Fraunhofer diffraction equation

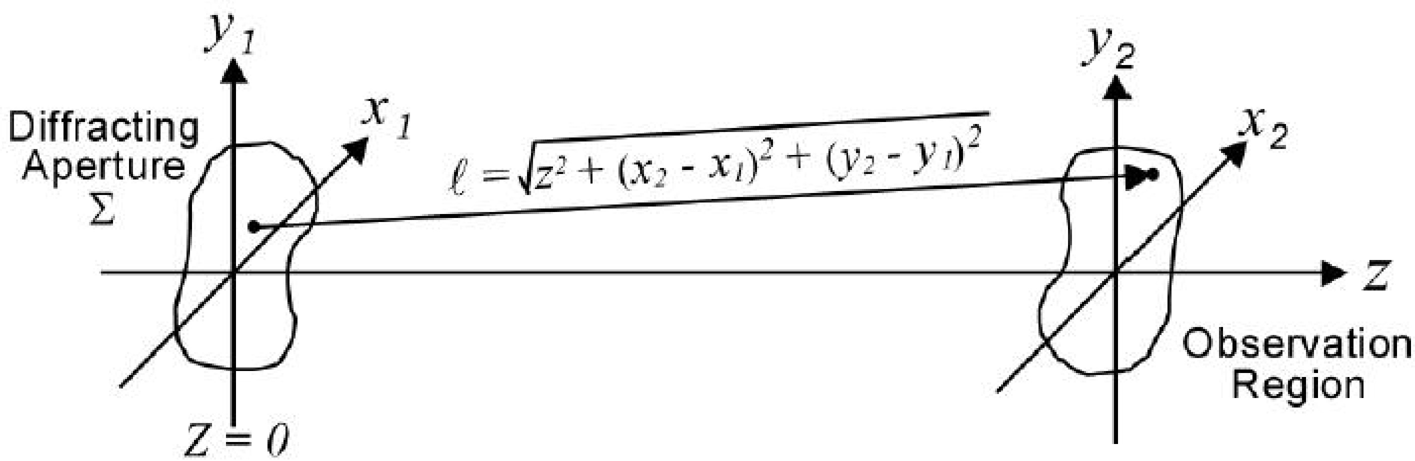


Figure 3-6. Geometrical relationship between diffracting aperture and observation space.

$$U_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}(x_2^2 + y_2^2)\right] \mathcal{F}\{U_1(x_1, y_1)\}\Big|_{\substack{\xi = x_2/\lambda z \\ \eta = y_2/\lambda z}}$$

$\mathcal{F}$ is Fourier Transform

Please note that Fourier transforming a function of x and y results in a function of spatial frequencies ξ and η, which must then be evaluated at $\xi = x_2 / \lambda z$ and $\eta = y_2 / \lambda z$.

Fraunhofer diffraction, continued

$$U_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}(x_2^2 + y_2^2)\right] \mathcal{F}\{U_1(x_1, y_1)\}\Big|_{\substack{\xi = x_2/\lambda z \\ \eta = y_2/\lambda z}}$$

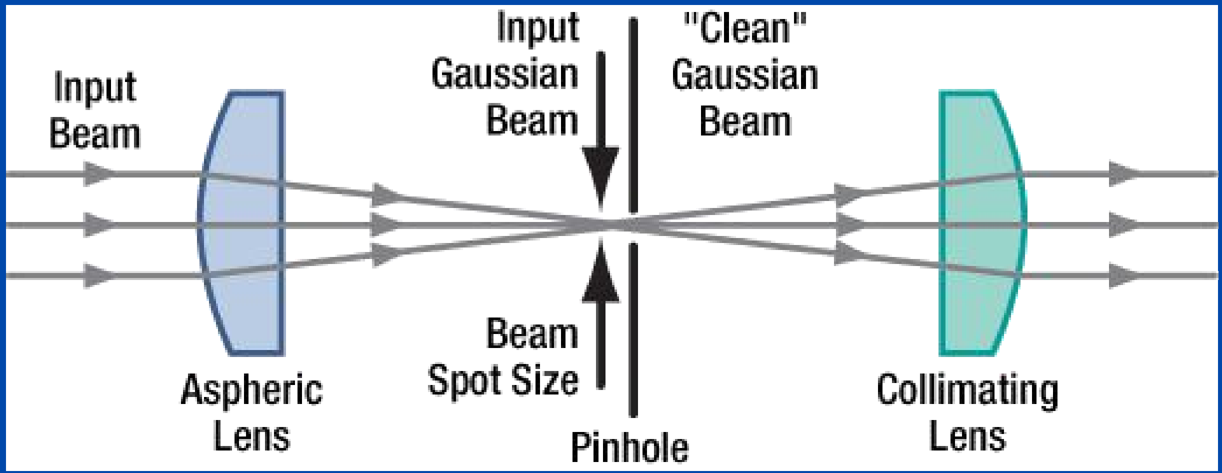
$\mathcal{F}$ is Fourier Transform

- In the "far field" (Fraunhofer limit) the diffracted field U_2 can be computed from the incident field U_1 by a phase factor times the Fourier transform of U_1

- "Image plane is Fourier transform of pupil plane"

Image plane is Fourier transform of pupil plane

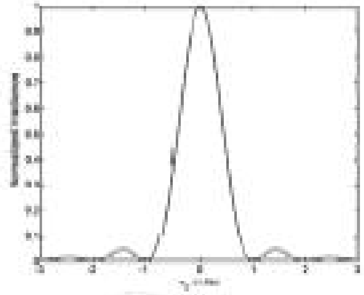
- Leads to principle of a "spatial filter"
- Say you have a beam with too many intensity fluctuations on small spatial scales
  - Small spatial scales = high spatial frequencies
- If you focus the beam through a small pinhole, the high spatial frequencies will be focused at larger distances from the axis, and will be blocked by the pinhole



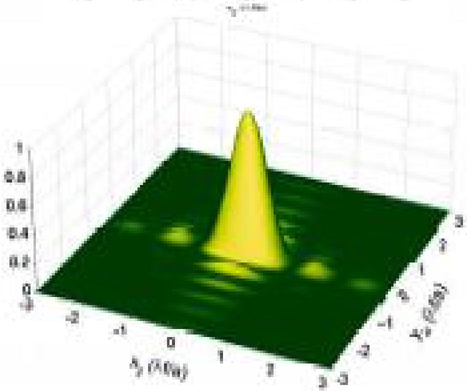
Rectangular Aperture

$$E_2(x_2, y_2) = \frac{E_o\, a^2 d^2}{\lambda^2 f^2}\, \mathrm{sinc}^2\left(\frac{x_2}{\lambda f / a}, \frac{y_2}{\lambda f / d}\right)$$

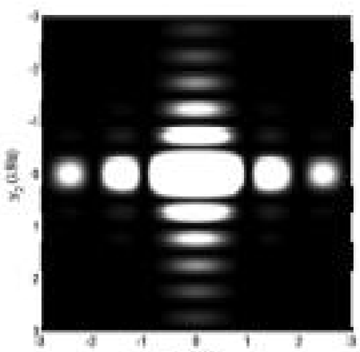
a.)



b.



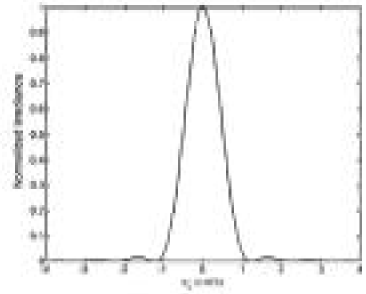
c.)



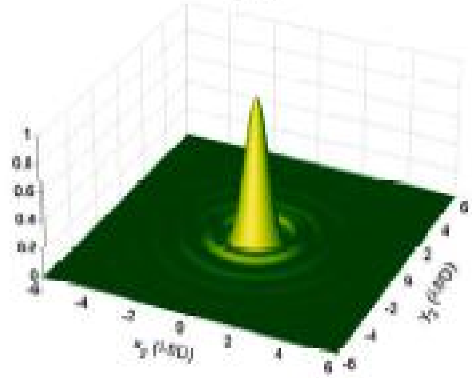
Circular Aperture

$$E_2(x_2, y_2) = E_o \left(\frac{\pi D^2}{4 \lambda f}\right)^2 \mathrm{somb}^2\left(\frac{r_2}{\lambda f / D}\right)$$

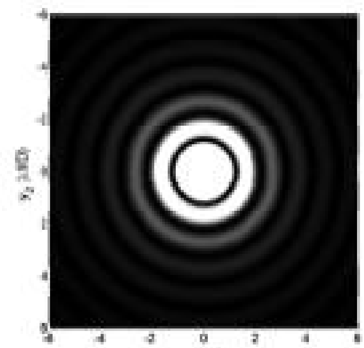
a.)



b



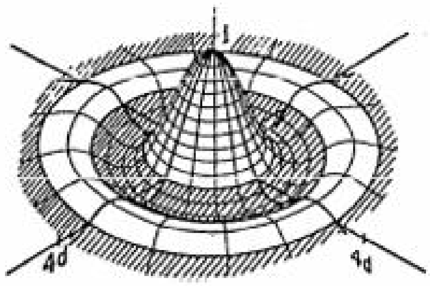
c.)

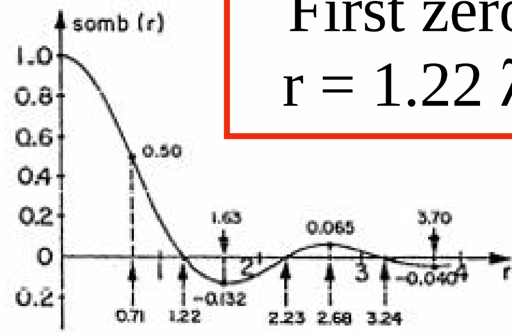


Details of diffraction from circular aperture

1) Amplitude

$$somb(r) = 2\,J_1(\pi r)/(\pi r)$$



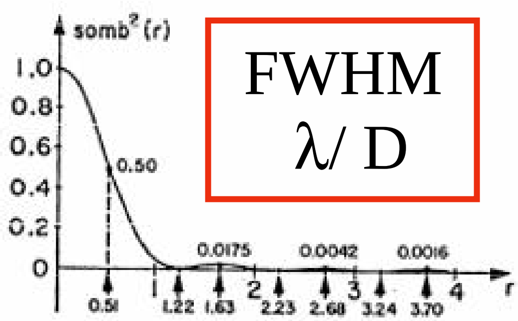


First zero at
$r = 1.22\ \lambda/D$

2) Intensity

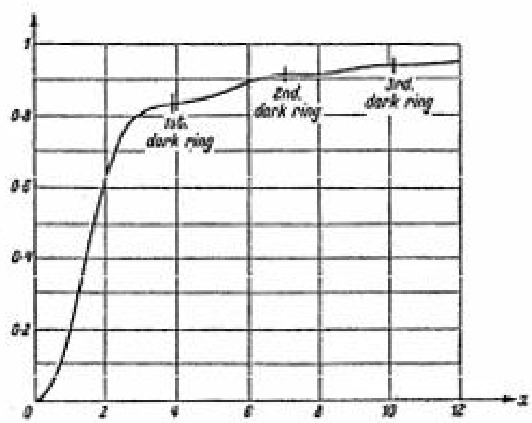
Airy Pattern

$somb^2(r)$

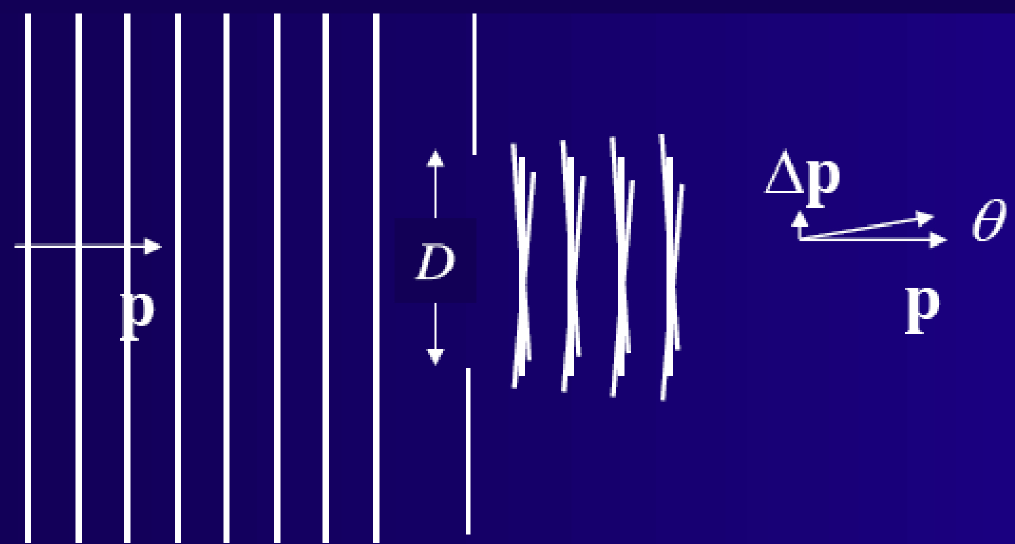


FWHM
λ/D

Fractional Encircled Energy



Heuristic derivation of the diffraction limit



Uncertainty principle

$$\Delta x \Delta p \simeq h$$

Photon momentum

$$p = h/\lambda$$

$$\Delta \mathbf{x} = \infty$$

$$\Delta \mathbf{p} = 0$$

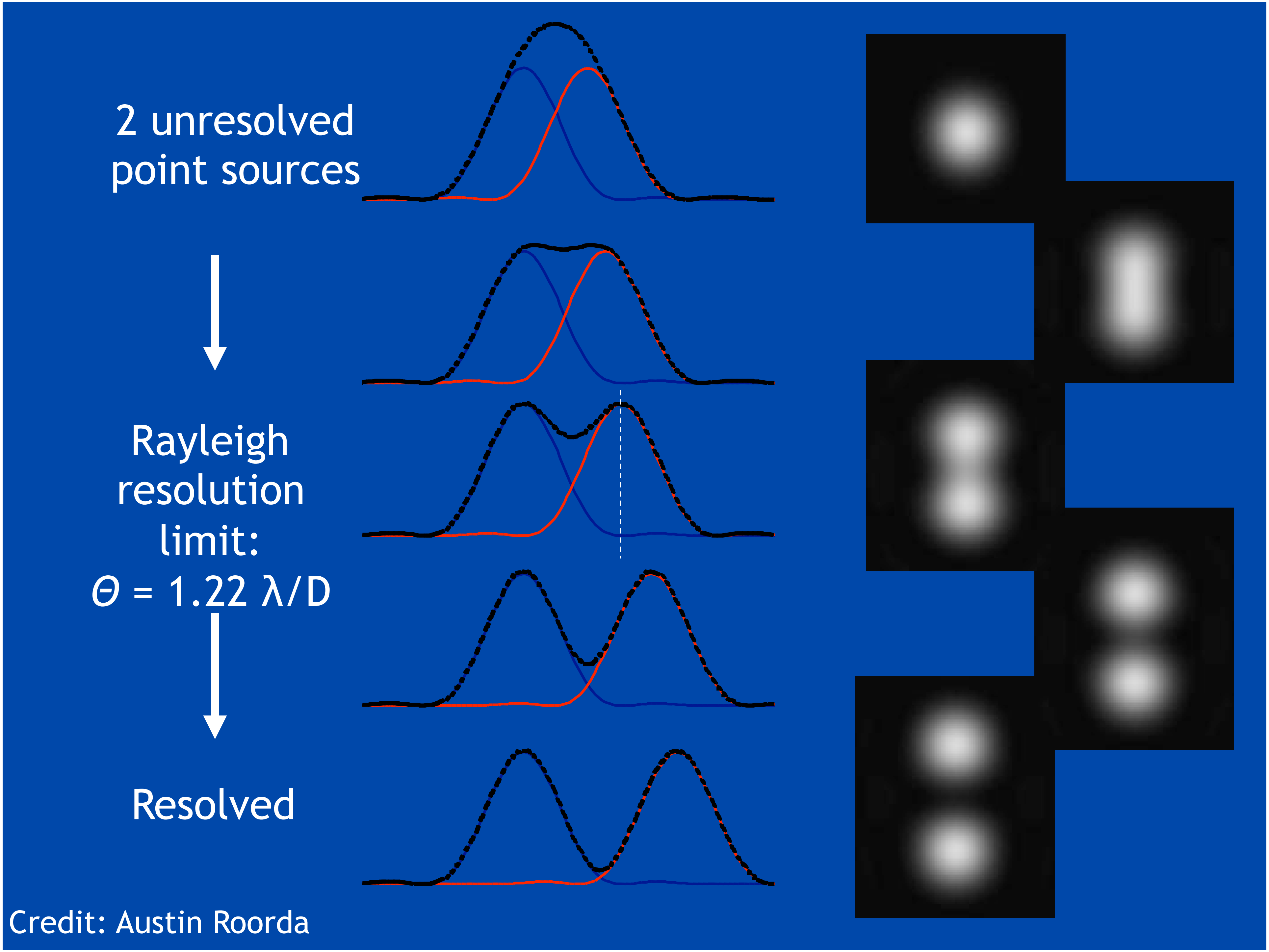
$$\Delta x = D$$

$$\Delta p = h/D$$

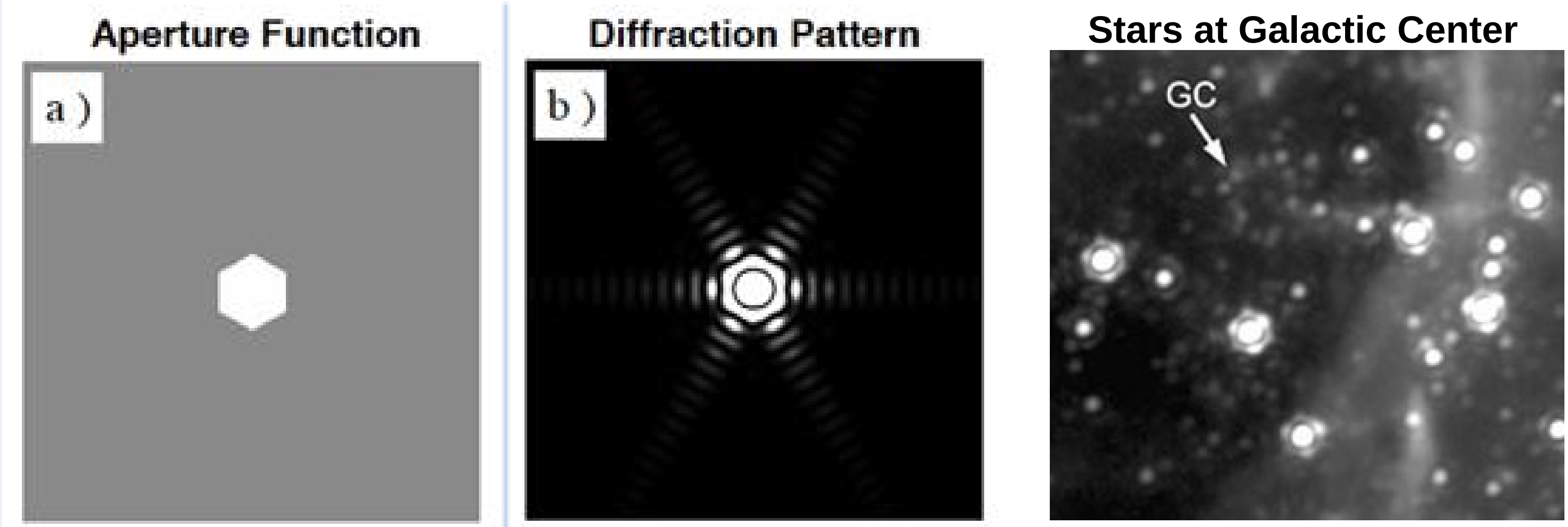
$$\Longrightarrow$$

Law of diffraction

$$\theta \simeq \Delta p / p = \lambda / D$$


2 unresolved point sources
Rayleigh resolution limit:
Θ = 1.22 λ/D
Resolved
Credit: Austin Roorda

Diffraction pattern from hexagonal Keck telescope



Ghez: Keck laser guide star AO

Conclusions:
In this lecture, you have learned …

- Light behavior is modeled well as a wave phenomena (Huygens, Maxwell)
- Description of diffraction depends on how far you are from the source (Fresnel, Fraunhofer)
- Geometric and diffractive phenomena seen in the lab (Rayleigh range, diffraction limit, depth of focus…)
- Image formation with wave optics